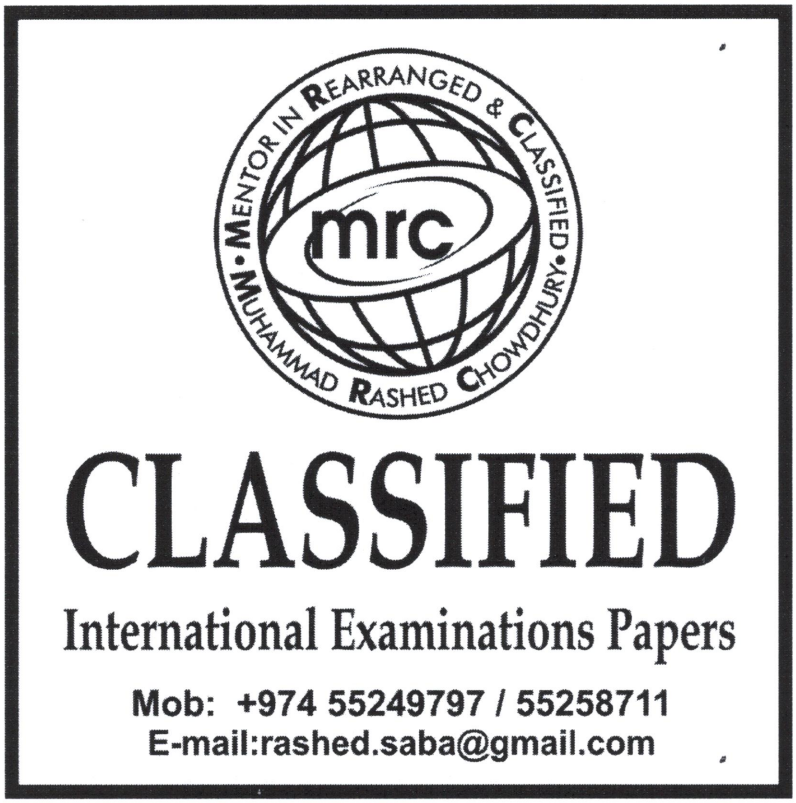


Pure Mathematics-1

TOPIC- Differentiation

Stationary Point (TP)

Maxima, Minima

DIFFERENTIATION-STATIONARY POINT(max,min)

0 1 A curve has equation $y = 8x + (2x - 1)^{-1}$. Find the values of x at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

7-11-13-5

DIFFERENTIATION-STATIONARY POINT(max,min)

0 2 Variables u, x and y are such that $u = 2x(y - x)$ and $x + 3y = 12$. Express u in terms of x and hence find the stationary value of u. [5]

15-12-4

DIFFERENTIATION-STATIONARY POINT(max,min)

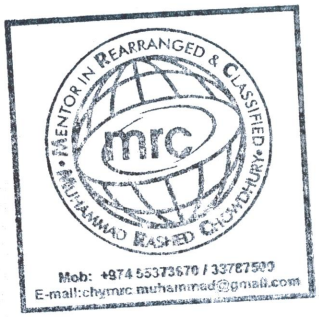
03 The non-zero variables x, y and u are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of u and determine whether this is a maximum or a minimum value. [7]

J- 13-136

4 The equation of a curve is $y = x^4 + 4x + 9$. N-9-11- kj

(i) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$. [3]



5 A curve has equation $y = 2x + \dfrac{1}{(x-1)^2}$. Verify that the curve has a stationary point at $x = 2$ and determine its nature. N-12-11-5 [5]

6 The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is 2000 cm^3.

(i) Express h in terms of x and show that the total surface area of the prism, A cm^2, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}.$$ [3]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Determine, showing all necessary working, the nature of this stationary value. [2]

7 The equation of a curve is $y = 2 + \frac{3}{2x-1}$.

N-16-12-7

N-16-12-Q-5P

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

8 A curve is such that

$$\frac{dy}{dx} = 2(3x+4)^{\frac{3}{2}} - 6x - 8.$$ N-12-13-8

(i) Find $\frac{d^2y}{dx^2}$. [2]

(ii) Verify that the curve has a stationary point when $x = -1$ and determine its nature. [2]

(iii) It is now given that the stationary point on the curve has coordinates $(-1, 5)$. Find the equation of the curve. [5]

9 A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1$ and $P(9, 5)$ is a point on the curve. J- 11-13-9

(i) Find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point on the curve. [3]

(iii) Find an expression for $\frac{d^2y}{dx^2}$ and determine the nature of the stationary point. [2]

(iv) The normal to the curve at P makes an angle of $\tan^{-1} k$ with the positive x-axis. Find the value of k. [2]

10 It is given that a curve has equation $y = f(x)$, where $f(x) = x^3 - 2x^2 + x$. J-12-11-10

(i) Find the set of values of x for which the gradient of the curve is less than 5. [4]

(ii) Find the values of $f(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

DIFFERENTIATION-STATIONARY POINT(max,min)

11 The volume of a solid circular cylinder of radius r cm is 250π cm^3. J-13-12-8

(i) Show that the total surface area, S cm^2, of the cylinder is given by

$$S = 2\pi r^2 + \frac{500\pi}{r}.$$ [2]

(ii) Given that r can vary, find the stationary value of S. [4]

(iii) Determine the nature of this stationary value. [2]

12 A curve is such that $\frac{dy}{dx} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$. The curve passes through the point $(4, \frac{2}{3})$.

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary point and determine its nature. [5]

13 The variables x, y and z can take only positive values and are such that J-11-11-6

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

(i) Show that $z = 3x + \dfrac{1200}{x}$. [1]

(ii) Find the stationary value of z and determine its nature. [6]

14 The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

(iii) Find the values of x at which the line $y = 6$ meets the curve. [3]

(iv) State the set of values of k for which the line $y = k$ does not meet the curve. [1]

15 A curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ and the point (9, 2) lies on the curve.

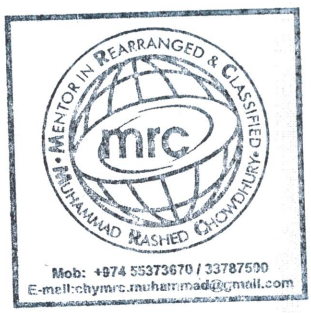
(i) Find the equation of the curve. [4]

(ii) Find the x-coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

DIFFERENTIATION-STATIONARY POINT(max,min)

16. A curve has equation $y = \frac{k^2}{x+2} + x$, where k is a positive constant. Find, in terms of k, the values of x for which the curve has stationary points and determine the nature of each stationary point. [8]

N-13-13-9



17 The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3.

(i) Show that the total surface area of the cuboid, A cm^2, is given by

$$A = 6x^2 + \frac{768}{x}.$$

(ii) Given that x can vary, find the stationary value of A and determine its nature.

18 The equation of a curve is $y = x^3 + px^2$, where p is a positive constant. J-15-11-9

(i) Show that the origin is a stationary point on the curve and find the coordinates of the other stationary point in terms of p. [4]

(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

19 The function f is defined for $x > 0$ and is such that $f'(x) = 2x - \frac{2}{x^2}$. The curve $y = f(x)$ passes through the point $P(2, 6)$.

N- 14-11-9

(i) Find the equation of the normal to the curve at P. [3]

(ii) Find the equation of the curve. [4]

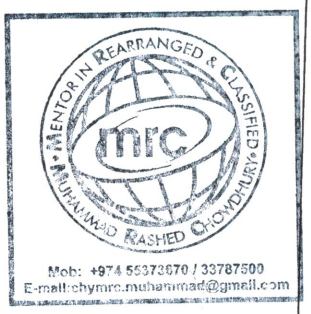
(iii) Find the x-coordinate of the stationary point and state with a reason whether this point is a maximum or a minimum. [4]

20 A curve has equation $y = f(x)$ and is such that $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$. 7-13-11-9

(i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = f(x)$ has stationary points. [4]

(ii) Find $f''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

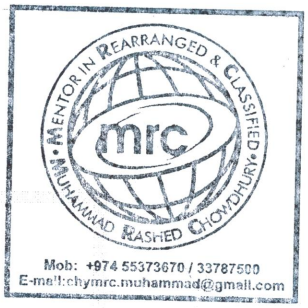
(iii) It is given that the curve $y = f(x)$ passes through the point $(4, -7)$. Find $f(x)$. [4]



2 1. A curve $y = f(x)$ has a stationary point at (3, 7) and is such that $f''(x) = 36x^{-3}$. N-14-13-8

(i) State, with a reason, whether this stationary point is a maximum or a minimum. [1]

(ii) Find $f'(x)$ and $f(x)$. [7]



22 A function f is defined for $x \in \mathbb{R}$ and is such that $f'(x) = 2x - 6$. The range of the function is given by $f(x) \geqslant -4$.

N-11-11-4

(i) State the value of x for which $f(x)$ has a stationary value. [1]

(ii) Find an expression for $f(x)$ in terms of x. [4]

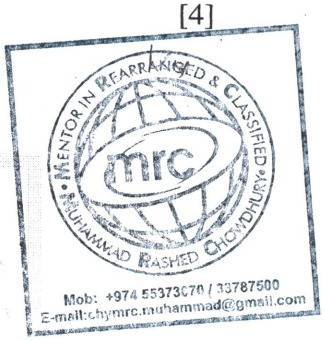
2 3 A curve $y = f(x)$ has a stationary point at $P(3, -10)$. It is given that $f'(x) = 2x^2 + kx - 12$, where k is a constant.

N-11-13-8

(i) Show that $k = -2$ and hence find the x-coordinate of the other stationary point, Q. [4]

(ii) Find $f''(x)$ and determine the nature of each of the stationary points P and Q. [2]

(iii) Find $f(x)$. [4]



24 The curve $y = f(x)$ has a stationary point at $(2, 10)$ and it is given that $f''(x) = \frac{12}{x^3}$.

N-15-12-9

A-15-12-DSP

(i) Find $f(x)$. [6]

(ii) Find the coordinates of the other stationary point. [2]

(iii) Find the nature of each of the stationary points. [2]

25 A curve is such that $\frac{d^2y}{dx^2} = \frac{24}{x^3} - 4$. The curve has a stationary point at P where $x = 2$.

N-14-12-10

(i) State, with a reason, the nature of this stationary point. [1]

(ii) Find an expression for $\frac{dy}{dx}$. [4]

(iii) Given that the curve passes through the point (1, 13), find the coordinates of the stationary point P. [4]