LEARNING OBJECTIVES:

Understanding

➔ Periodic motions and Simple harmonic oscillations
➔ Concepts of Time period, frequency, amplitude, displacement, and phase difference
➔ Conditions for simple harmonic motion

Applications and skills

➔ Sketching and interpreting graphs of simple harmonic motion examples
➔ Qualitatively describing the energy changes/ transformations taking place during one cycle of an oscillation
➔ Qualitatively explain conservation principle of total energy of bodies having simple harmonic motion graphically
➔ Apply simple harmonic motion in everyday life

➔Equations:

F = - kx Time period (T) $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ $as\ \omega = \sqrt{\frac{k}{m}}$ Frequency (n) $= \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	$\theta = \omega t$ $y = y_0 \sin \omega t$ and $x = x_0 \cos \omega t$ or, $x = x_0 \sin \omega t$	$v = x_0 \omega \cos \omega t$ $v = A \omega \cos \omega t$ $v = \pm\omega\sqrt{x_0^2 - x^2}$	$a = - x_0 \omega^2 \sin \omega t$ $a = -\omega^2 x$
$E_k = \frac{1}{2} mA^2\omega^2 COS^2\omega t$ $E_k = \frac{1}{2} KA^2 \ COS^2\omega t$ $E_k = \frac{1}{2} m\omega^2(x_0^2 - x^2)$	$E_P = \frac{1}{2} KA^2 sin^2\omega t$ $E_P = \frac{1}{2} m\omega^2 x^2$	$E_T = \frac{1}{2} KA^2(sin^2\omega t + cos^2\omega t)$ $E_T = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2} KA^2$	$h = R\left(\frac{T_1}{T} - 1\right)\ldots$ $h = R\left(\sqrt{\frac{g}{g_1}} - 1\right)$

Periodic Motion:

A motion, which repeat itself over and over again after a regular interval of time is called a periodic motion and the fixed interval of time after which the motion is repeated is called period of the motion.

Examples: Revolution of earth around the sun (period one year).

Special periodicity:

Oscillatory (HARMONIC) or Vibratory Motion:

The motion in which a body/particle moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. That is it remains half of its time period in one direction and the rest of the time period remains in other direction.

Oscillatory motion is also called as harmonic motion.

Example: The motion of the pendulum of a wall clock, motion of strings of a guitar, oscillation of air-particles during the transmission of sound etc.

Harmonic and Non-harmonic Oscillation.

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function *(i.e.* sine or cosine function).
Example: $y = a \sin\omega t$ or $y = a \cos\omega t$.

Non-harmonic oscillation is that oscillation which cannot be expressed in terms of single harmonic function. *Example:* $y = a \sin\omega t + b \sin 2\omega t$.

Some Important Definitions.

Amplitude: Its deviation from the mean position.
[**Amplitude** *A* is the maximum displacement of a wave from its rest position.]

Time period: It is the least interval of time after which the periodic motion of a body repeats itself.
S.l. units of time period are second.
[**Period** *T* is the time that it takes for one complete wavelength to pass a fixed point or for a particle to undergo one complete oscillation.]

Frequency: It is defined as the number of periodic motions executed by body per second.
S.l unit of frequency is hertz (Hz).
Frequency *f* is the number of vibrations per second performed by the source of the waves and so is equivalent to the number of crests passing a fixed point per second.

Angular Frequency: Angular speed, ω is the rate of change of angle with time and is also called **angular frequency**.
It is measured in radians per second (rad s^{-1}).
This quantity is important when we deal with simple harmonic motion because there is a very close relationship between circular motion and SHM.

Simple Harmonic Motion:

Simple harmonic motion is a special type of periodic motion, in which restoring force (acceleration) is opposite and proportional to displacement of the particle from mean position.

$$F \propto -x \quad or,\ F = -kx$$

where k is known as force constant. Its S.l. unit is Newton/meter and dimension is $[MT^{-2}]$.

The main characteristics of SHM are:

- **the period and amplitude are constant**
- **the period is independent of the amplitude**
- a special type of force(restoring) acting on a particle having simple harmonic motion
- acceleration of the particle and magnitude of the force acting on it is proportional to the displacement
- direction of the acceleration and force acting on the particle always remains towards the equilibrium, i.e., in the opposite direction of displacement.
- **the displacement, velocity and acceleration are sine or cosine functions of time**.

Obviously, all the periodic motions do not have those characteristics. So, all simple harmonic motions are periodic motions but all periodic motions are not simple harmonic motions.

Circular motion and SHM

In mathematical terms the demonstration in figure 1 is equivalent to projecting the two-dimensional motion of a point onto the single dimension of a line.

Imagine a point P rotating around the perimeter of a circle with a constant angular speed, **ω**. The radius of the circle *r* joins P with the centre of the circle O.

At time $t = 0$ the radius is horizontal and at time *t* it has moved through an angle θ radians.

For constant angular speed, $\boldsymbol{\omega} = \frac{\boldsymbol{\theta}}{\mathbf{t}}$ rearranging this gives $\theta = \omega t$.

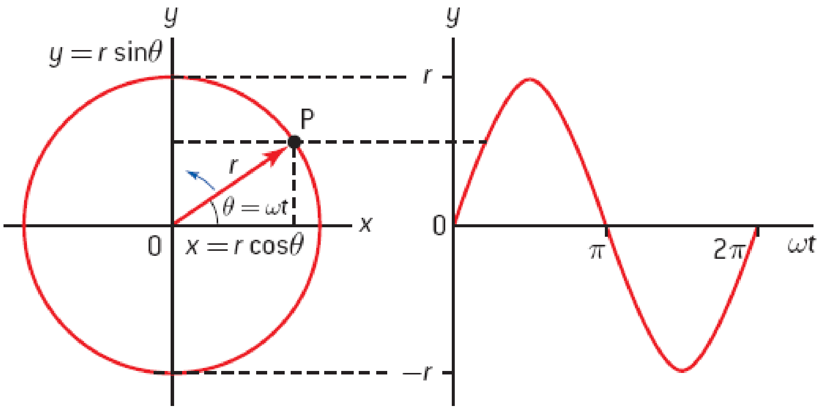
Projecting P onto the *y*-axis gives the vertical component of *r* as $r \sin \theta$.

Projecting P onto the *x*-axis gives the horizontal component of *r* as $r \cos \theta$.

The variation of the vertical component with time or angle is shown on the right of figure 2 and takes the form of a sine curve.

Because the rate of rotation is constant, the angle θ or ωt is proportional to time.

If we drew a graph of *y* against *t* the quantities 2π and π would be replaced by *T* and $T/2$ respectively.



Projection of circular motion on to a vertical line.

The equations of the projections are $y = y_0 \sin \omega t$ and $x = x_0 \cos \omega t$.

Here y_0 and x_0 are the maximum values of *y* and *x*, **which in this case are identical to *r*. These are the amplitudes of the motion.**

This is very useful in analyzing SHM. In SHM the displacement, velocity and acceleration all vary sinusoid ally with time. Thus, the projection of circular motion on the vertical or horizontal takes the same shape as SHM.

Phase: It is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position (equilibrium).

$Y = a \sin\theta = a \sin(\omega t + \phi_0)$ here $\theta = \omega t + \phi_0$, phase of vibrating particle.

(i) Initial phase or epoch: It is the phase of a vibrating particle at $t = 0$.

(ii) Same phase: Two vibrating particle are said to be **in same phase**, if the phase difference between them is an even multiple of (**π**) or path difference is an even multiple of (λ/2) or time interval is an even multiple of (T/2).

(iii) Opposite phase: Opposite phase means the phase difference between the particles is an odd multiple of (**π**) or the path difference is an odd multiple of λ or the time interval is an odd multiple of (T/2).

(iv)Phase difference: If two particles perform S.H.M and their equation are
$y_1 = a \sin(\omega t + \phi_1)$ and $y_2 = a \sin(\omega t + \phi_2)$
then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$
Period T is equivalent to 360° or 2π radians,
so $\frac{T}{2}$ is equivalent to 180° or π radians
and $\frac{T}{4}$ is equivalent to 90° or $\frac{\pi}{2}$ radians.

When the phase difference is 0 or T then two systems are said to be oscillating **in phase**.
The amount by which one curve is shifted forward relative to another curve is called the phase difference between the two curves. Technically, the phase difference is described in terms of an angle φ, where:
The three graphs of Figure **4.8** all show simple harmonic oscillations with the same amplitude and period (and hence frequency). There is however a **phase difference** between them. The blue curve is the red curve shifted forward by some amount. And the purple curve is the red curve shifted forward by an even greater amount.

the

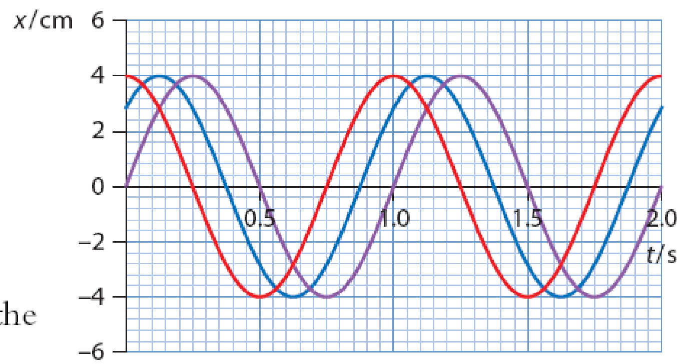


Figure 4.8 Three graphs of simple harmonic oscillations with a phase difference between them.

$$\varphi = \frac{\text{shift}}{T} \times 360°$$

x/c

Relative to the red curve, the blue curve is shifted by 0.125 s and the period is 1.00 s, so the phase difference is:

$$\varphi = \frac{0.125}{1.00} \times 360° = 45° \ (\text{or } \frac{\pi}{4} \text{ radians})$$

Relative to the purple curve, the purple curve is shifted by 0.250 s and the period is again 1.00 s, so the phase difference is:

$$\varphi = \frac{0.250}{1.00} \times 360° = 90° \ (\text{or } \frac{\pi}{2} \text{ radians})$$

The relationship between displacement, velocity, and acceleration

Graphically starting from the displacement–time curve, we could derive the velocity–time and acceleration–time curves from the gradients.

But another technique is to **differentiate the equations with respect to time** (differentiation is equivalent to finding the gradient).

We have seen from the **comparison with circular motion that the displacement** takes the form:

Displacement:

$x = x_0 \sin \omega t$ **or** $x = x_0 \cos \omega t$ depending on when we start timing.

It really doesn't matter whether the projection is onto the x-axis or the y-axis – therefore we can use x and y **interchangeably.**

Let's start with $x = x_0 \sin \omega t$

Velocity:

This means that the velocity is given by

$v\left(=\frac{dx}{dt}\right) = x_0 \omega \cos \omega t$, where x_0 is the amplitude and ω the angular frequency.

The maximum value **that cosine can take is 1** so the maximum velocity is $v_0 = x_0 \omega$ thus making the equation for the velocity at time t become

$v = v_0 \cos \omega t$

The velocity equation

We have already seen that

$v = v_0 \cos \omega t = x_0 \omega \cos \omega t$

We know, $\cos\omega t = \pm\sqrt{1 - \sin^2\omega t}$

But $\sin^2\omega t = \left(\frac{x}{x_0}\right)^2$

So $v = \pm x_0 \omega \sqrt{1 - \left(\frac{x}{x_0}\right)^2} = \pm\omega\sqrt{x_0^2 - x^2} = \pm\omega\sqrt{A^2 - x^2}$ $\quad$ ($x_0 = A$)

At x=A, at the end of amplitude	At x=0, at equilibrium position
$v_{min} =$	$v_{max} =$

The equation is useful for finding the velocity at a particular position when you know the amplitude and period (or frequency or angular frequency) – you don't need to know the time being considered.

Acceleration:

We know that the acceleration will be the gradient of a velocity–time graph so we have

$a\left(=\frac{dv}{dt}\right) = -v_0 \omega \sin \omega t = -x_0 \omega^2 \sin \omega t$

As for cosine, the maximum value that sine can take is 1 so

$a_0 = v_0 \omega = x_0 \omega \omega = x_0 \omega^2$ **giving** $\qquad a = -a_0 \sin \omega t$

Comparing the equations $x = x_0 \sin \omega t$ **and** $a = -x_0 \omega^2 \sin \omega t$ we can see a common factor of $x_0 \sin \omega t$ meaning that

$$a = -x_0 \omega^2 \sin \omega t$$
$$a = -\omega^2 x$$

we saw that $a = -kx$ for SHM.

So the constant k must actually be ω^2 (the angular frequency squared).

The acceleration equation

We have already seen that

$a = -v_0\,\omega \sin\omega t = -x_0\,\omega^2 \sin\omega t = -A\,\omega^2 \sin\omega t$

But $sin\omega t = \frac{x}{x_0}$

So, $a = -x_0\omega^2(\frac{x}{x_0}) = -\omega^2 x$

Negative sign indicates that acceleration and displacement are opposite to eachother.

At x=A, at the end of amplitude	*At x=0, at equilibrium position*
a_{max} =	a_{min} =

Comparative Study of Displacement, Velocity and Acceleration:

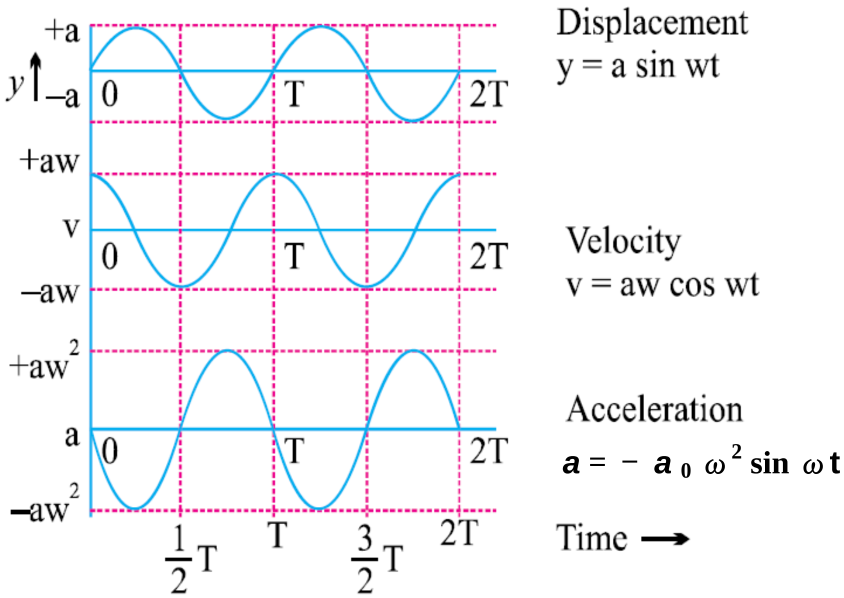
Displacement $y = a\sin\omega t$

Velocity $v = a\omega\cos\omega t$

$$v = a\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

Acceleration, $a = -a\omega^2 \sin\omega t$

$$\omega t = a\omega^2 \sin(\omega t + \pi)$$



(i) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

(ii) The velocity amplitude is ω times the displacement amplitude

(iii) The acceleration amplitude is ω^2 times the displacement amplitude

(iv) In S.H.M. the velocity is ahead of displacement by a phase angle $\pi/2$.

(v) In S.H.M. the acceleration is ahead of velocity by a phase angle $\pi/2$.

(vi) The acceleration is ahead of displacement by a phase angle of π.

Various physical quantities in S.H.M. at different position:

Physical quantities	Equilibrium position ($y = 0$)	Extreme Position ($y = \pm a$)
Displacement $y = a\sin\omega t$	Minimum (Zero)	Maximum (a)
Velocity $v = \omega\sqrt{a^2 - y^2}$	Maximum ($a\omega$)	Minimum (Zero)
Acceleration $A = -\omega^2 y$	Minimum (Zero)	Maximum ($\omega^2 a$)

Energy in SHM systems

We have seen that, in a simple pendulum, there is energy interchange between gravitational potential and kinetic; in a horizontal mass–spring system the interchange is between elastic potential and kinetic energy.
Because each system involves kinetic energy we will focus on this form of energy and bear in mind that the potential energy will always be the difference between the total energy and the kinetic energy at a particular time.
The total energy will be equal to the maximum kinetic energy.
we know that the kinetic energy of an object of mass m, moving at velocity v, is given by

$$E_k = \frac{1}{2}mv^2$$

We also know that the equation for the velocity at a particular position is

$v = \pm\omega\sqrt{x_0^2 - x^2}$	$v = x_0 \omega \cos\omega t = A\ \omega \cos\omega t$

Combining these equations gives the kinetic energy at displacement x:

$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$ Or. $E_k = \frac{1}{2}m\omega^2(A^2 - x^2)$	$E_k = \frac{1}{2}mA^2\omega^2 COS^2\omega t$ $E_k = \frac{1}{2}KA^2\ \ COS^2\omega t$ Since, $K=m\omega^2$

The maximum velocity is achieved when $x = 0$, i.e. as the mass moves past its equilibrium position. Here there is no extension, so the elastic potential energy is zero.
This tells us that the maximum kinetic energy will be given by

$E_{max} = \frac{1}{2}m\omega^2 x_0^2$ $E_{max} = \frac{1}{2}Kx_0^2 = E_{max} = \frac{1}{2}KA^2$ at $x = 0$ it has kinetic energy only.	$E_{max} = \frac{1}{2}KA^2$ As maximum value of $COS^2\omega t$ is 1.

and this must be the total energy (**when the potential energy is zero**) so we can say

$$E_T = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}KA^2$$

The potential energy at any position will be the difference between the total energy and the kinetic energy so

$$E_P = E_T - E_K = \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$E_P = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}Kx^2$	$E_P = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}KA^2 sin^2\omega t$

At the extremes of the motion, $x = \pm A$ and $v = 0$, so the kinetic energy is zero.
Thus at $x = \pm A$ the system has elastic potential energy only
the maximum potential energy will be given by

$E_{P(max)} = \frac{1}{2}Kx_0^2 = E_{P(max)} = \frac{1}{2}KA^2$ $x = \pm A$ the system has elastic potential energy only,	$E_{P(max)} = \frac{1}{2}KA^2$ As maximum value of $sin^2\omega t$ is 1.

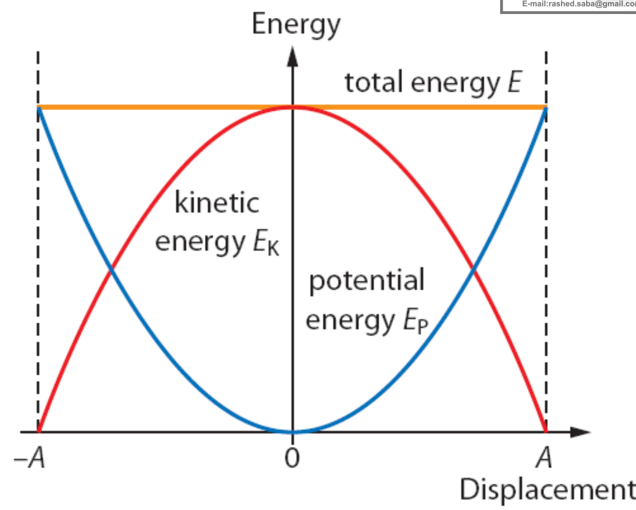
ALTERNATIVE WAY:

$E_T = E_P + E_K$

$= \frac{1}{2}KA^2 sin^2\omega t + \frac{1}{2}KA^2\omega^2 COS^2\omega t$

$E_T = \frac{1}{2}KA^2(sin^2\omega t + cos^2\omega t) = \frac{1}{2}KA^2$

Figure : Graphs showing the variation with displacement of the potential energy and kinetic energy of a particle on a spring. The total energy is a horizontal straight line.



The total energy of the system is then:

$E = E_P + E_K$

In the absence of frictional and other resistance forces, this total energy is conserved, and so E = constant.

At intermediate points the system has both forms of energy: elastic potential energy and kinetic energy. During an oscillation, we therefore have **transformations** from one form of energy into another.

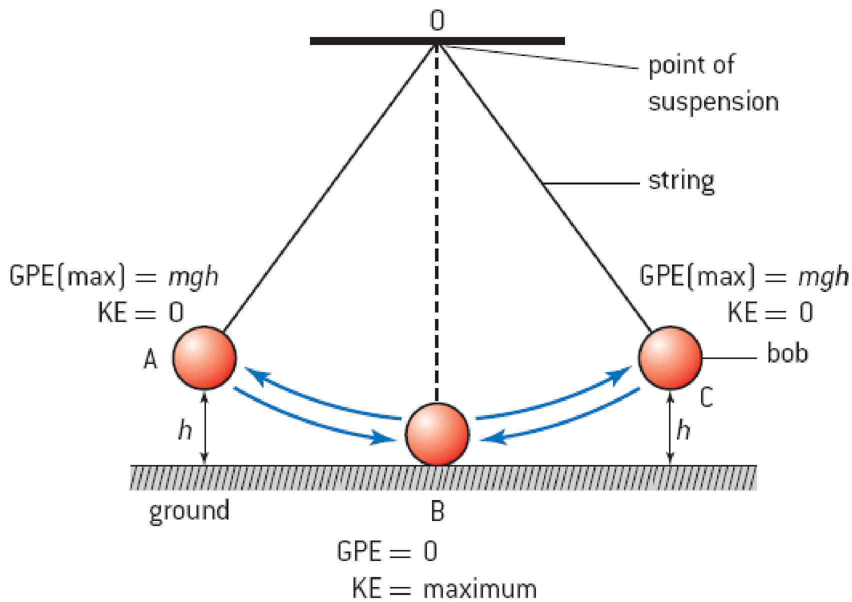
This allows us to write:

$E = E_P + E_K = (E_K)_{max} = (E_P)_{max}$

Energy changes in SHM:

Let's think of the motion of the simple pendulum of figure 11 as an example of a system which undergoes simple harmonic motion.

For a pendulum to oscillate simple harmonically the string needs to be long and to make small angle swings (less than 10°). The diagram is, therefore, not drawn to scale. Let's imagine that the bob just brushes along the ground when it is at its lowest position. When the bob is pulled to position A, it is at its highest point and has a maximum gravitational potential energy (GPE). As the bob passes through the rest position B, it loses all the GPE and gains a maximum kinetic energy (KE). The bob now starts to slow down and move towards position C when it briefly stops, having regained all its GPE. In between A and B and between B and C the bob has a combination of KE and GPE.



In a damped system over a long period of time the maximum height of the bob and its maximum speed will gradually decay. The energy gradually transfers into the internal energy of the bob and the air around it.

Note

- **The total energy is always the sum of the kinetic and potential energies.**
- **The graphs (unlike sine and cosine) never become negative.**
- **The period of the energy change is half that of the variation with time of displacement, velocity, or acceleration.**
- **The frequency of the energy change is twice that of the variation with time of displacement, velocity, or acceleration.**

Simple harmonic systems

1. The simple pendulum

The simple pendulum represents a straightforward system that oscillates with SHM when its amplitude is small. When the pendulum bob (the mass suspended on the string) is displaced from the rest position there is a component of the bob's weight that tends to restore the bob to its normal rest or equilibrium position. A condition of a system oscillating simple harmonically is that there is a restoring force that is proportional to the displacement from the equilibrium position – this is, in effect, tying in with the equation defining SHM because

$F = ma$ and $a = -\omega^2 x$ this means that $F = -m\omega^2 x$.

Figure 4 shows the forces acting on a pendulum bob. The bob is in equilibrium along the radius when the tension in the string F_t equals the component of the weight in line with the string ($= mg\cos\theta$). The component of the weight perpendicular to this is not in equilibrium and provides the restoring force.

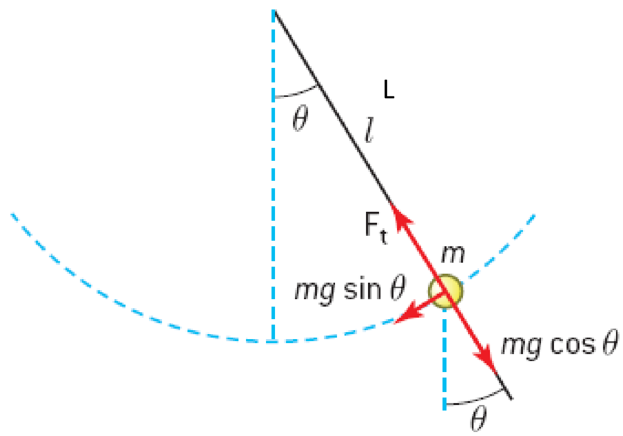


Figure 4 Restoring force for simple pendulum.

So the restoring force must be equal to the mass multiplied by the acceleration according to Newton's second law of motion:

$mg\sin\theta = ma$

For a small angle $\sin\theta \approx \theta \approx \frac{X}{L}$, L=I+r =effective length

rearranging gives $-m\left(\frac{g}{L}\right)x = ma$

(The minus sign is because the displacement (to the right) is in the opposite direction to the acceleration (to the left) in figure 4.

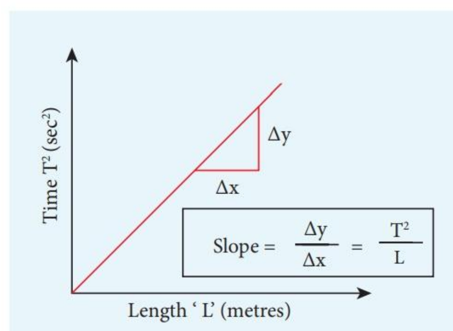
Cancelling m $\quad a = -\left(\frac{g}{L}\right)x$

this compares with the defining equation for SHM $a = -\omega^2 x$ leading to

$$\omega^2 = \frac{g}{L}$$

As the periodic time for SHM is given by $T = \frac{2\pi}{\omega}$

This shows that $T = 2\pi\sqrt{\frac{L}{g}}$ $\quad or, g = 4\pi^2\frac{L}{T^2}$



$slope = \frac{\Delta y}{\Delta x} = \frac{T^2}{L}$; 1/slope = L/T²

Graphical method:

$$T^2 \infty L$$

So, the L-T ^{2} graph will be a straight line passing through the origin.

$$g = 4\pi^2\frac{L}{T^2}$$

By the reciprocal of the slope we can determine 'g' easily.

Precautions:

Note

- The period of a simple pendulum is independent of the mass of the pendulum.
- The period of a simple pendulum is independent of the amplitude of the pendulum.
- l is the length of the pendulum from the point of suspension to the centre of mass of the bob.
- The equation only applies to small oscillations (swings making angle of less than 10° with the rest position).

Determination of height of a mountain by simple pendulum:

We can determine the height of a mountain or a tall building from earth's surface by a simple pendulum [Figure].

For this, we need to measure the time periods of oscillation at the bottom and at the top of the mountain.

Let the two time periods be T and T_1 and if g and g_1 be the values of the acceleration due to gravity at the bottom and at the top of the mountain respectively, then,

$$\frac{T_1}{T} = \sqrt{\frac{g}{g_1}}$$

Nov from Newton's law of gravitation, at the **bottom of the mountain,**

$$g = \frac{GM}{R^2} \dots \dots (1)$$

and at the top of the mountain,

$$g_1 = \frac{GM}{(R+h)^2} \dots \dots (2)$$

Here, M = mass of the earth, R = radius of the earth and h = height of the Mountain.

Dividing equation (1) by equation (2) we get,

$$\frac{g}{g_1} = \frac{(R+h)^2}{R^2}$$

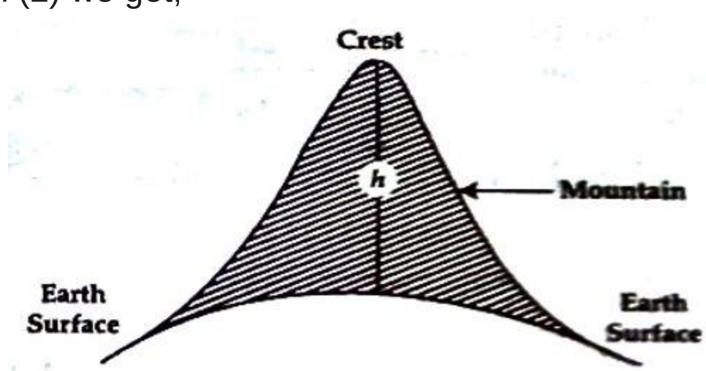
or, $\frac{R+h}{R} = \sqrt{\frac{g}{g_1}} = \frac{T_1}{T}$

or, $1 + \frac{h}{R} = \frac{T_1}{T}$

or, $\frac{h}{R} = \frac{T_1}{T} - 1$

or, $\boldsymbol{h = R\left(\frac{T_1}{T} - 1\right)} \dots \quad \dots \quad \dots$

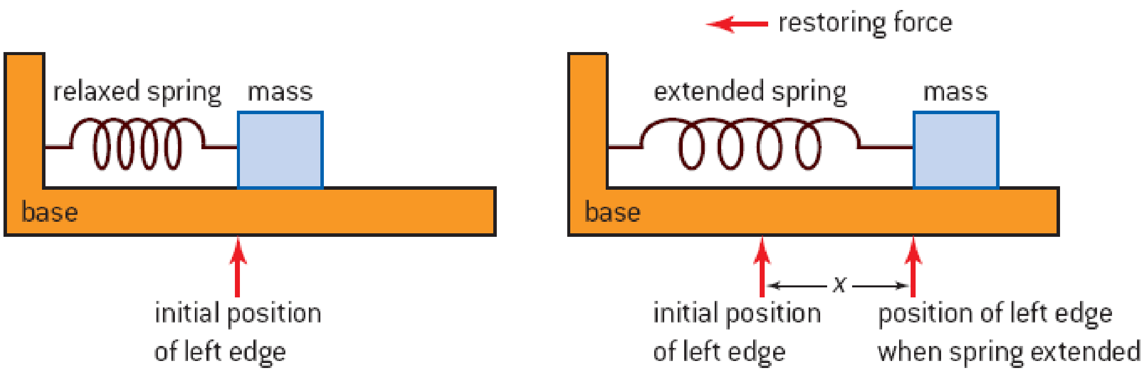
(3)OR, $\boldsymbol{h = R\left(\sqrt{\frac{g}{g_1}} - 1\right)} \dots$



In equation, (3), R, T and T_1 is known, so h can be find out.

2. Mass-spring system:

We have focused on a simple pendulum as being a very good approximation to SHM. A second system which also behaves well and gives largely un damped oscillations is a mass–spring system.
We will consider a mass being oscillated horizontally by a spring (see figure 5);
this is more straightforward than taking account of including the effects of gravity experienced in vertical motion.
We will assume that the friction between the mass and the base is negligible. The mass, therefore, exchanges elastic potential energy (when it is fully extended and compressed) with kinetic energy (as it passes through the equilibrium



Restoring force for mass-spring system.
When a spring (having spring constant k) is extended by x from its equilibrium position there will be a restoring force acting on the mass given by $F = -kx$ (the force is in the opposite direction to the extension).

Using Newton's second law
$ma = -kx$
which can be written as

$$a = -\left(\frac{k}{m}\right)x$$

this compares with the defining equation for SHM

$$a = -\omega^2 x$$

$$\omega^2 = \frac{k}{m}$$

As the periodic time for SHM is given by

$$T = \frac{2\pi}{\omega}$$

this shows that

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In general *m* is called inertia factor and *k* is called spring factor.

When the spring is compressed the quantity x represents the compression of the spring. When the mass is to the left of the equilibrium position the compression is positive but the restoring force will be negative. This,therefore, leads to the same outcomes as for extensions.

Note

- The period of the mass-spring system is independent of amplitude (for small oscillations).
- The period of the mass-spring system is independent of the acceleration of gravity.

In order to perform SHM an object must have a restoring force acting on it.

• The magnitude of the force (and therefore the acceleration) is proportional to the displacement of the body from a fixed point.
• The direction of the force (and therefore the acceleration) is always towards that fixed point.
Focusing on the loaded spring, the "fixed point" in the above definition is the equilibrium position of the mass – where it was before it was pulled down.
The forces acting on the mass are the tension in the spring and the pull of gravity (the weight). In the equilibrium position the tension will equal the weight but above the equilibrium the tension will be less and the weight will pull the mass downward; below the equilibrium position the tension will be greater than the weight and this will tend to pull the mass upwards.
The difference between the tension and the weight provides the restoring force – the one that tends to return the mass to its equilibrium position.
We can express the relationship between acceleration a and displacement x as:
$a \propto -x$
which is equivalent to
$a = -kx$
This equation makes sense if we think about the loaded spring. When the spring is stretched further the displacement increases and the tension increases. Because *force* = *mass* × *acceleration*, increasing the force increases the acceleration. The same thing applies when the mass is raised but, in this case, the tension decreases, meaning that the weight dominates and again the acceleration will increase. So although we cannot prove the proportionality (which we leave for HL) the relationship makes sense.
The minus sign is explained by the second bullet point "the acceleration is always in the opposite direction to the displacement". So choosing upwards as positive when the mass is above the equilibrium position, the acceleration will be downwards (negative). When the mass is below the equilibrium position (negative), the (net) force and acceleration are upwards (positive). The force decelerates the mass as it goes up and then accelerates it downwards after it stopped.
Figure: is a graph of a against x. We can see that $a = -kx$. takes the form $y = mx + c$ with the gradient being a negative constant.

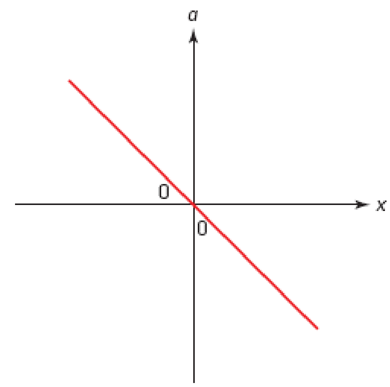


Figure : Acceleration displacement graph.

Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy:
Potential energy(or, elastic potential energy) and Kinetic energy

(1) Potential energy: $U = \frac{1}{2}m\omega^2a^2\sin^2\omega t$

(i) $U_{max} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2a^2$ When, $y = \pm a;\ \omega t = \frac{\pi}{2}; t = \frac{T}{4}$

(ii) $U_{min} = 0$ when, $y = 0;\ \omega t = 0; t = 0$

(2) Kinetic energy: $K = \frac{1}{2}m\omega^2a^2\cos^2\omega t \quad or, K = \frac{1}{2}m\omega^2(a^2 - y^2)$

(i) $K_{max} = \frac{1}{2}m\omega^2a^2$ when, $y = 0;\ \omega t = 0; t = 0$

(ii) $K_{min} = 0$ when, $y = \pm a;\ \omega t = \frac{\pi}{2}; t = \frac{T}{4}$

(3) Total energy : Total mechanical energy
= Kinetic energy + Potential energy

$$E = \frac{1}{2}m\omega^2a^2$$

Total energy is **not a position function** *i.e.* it always remains constant.

Problem: An object performs SHM with a period of 0.40 s and has amplitude of 0.20 m. The displacement is zero at time zero. Calculate:

a) the maximum velocity
b) the magnitude of the velocity after 0.10 s
c) the maximum acceleration of the object.

Problem: An object oscillates simple harmonically with frequency 60 Hz and amplitude 25 mm. Calculate the velocity at a displacement of 8 mm.

Problem:

Figure Graph showing the variation with displacement of the kinetic energy of a particle.

The graph in Figure shows the variation with displacement of the kinetic energy of a particle of mass 0.40 kg performing SHM at the end of a spring.

a Use the graph to determine:

i the total energy of the particle

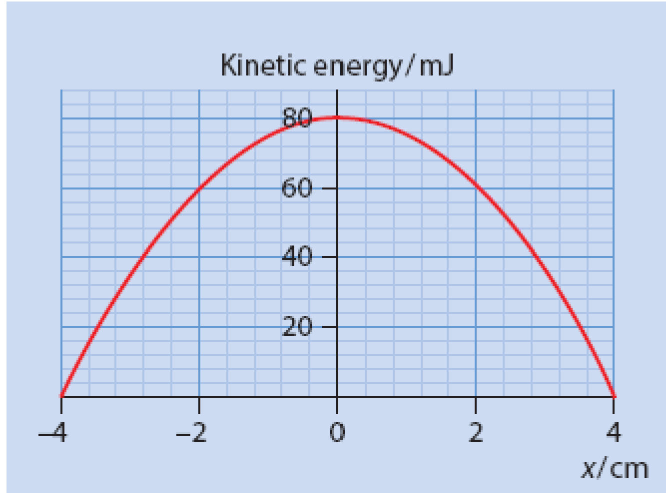
ii the maximum speed of the particle

iii the amplitude of the motion

iv the potential energy when the displacement is 2.0 cm.

b On a copy of the axes, draw the variation with displacement of the potential energy of the particle.

Answer :



Problem: The mass–spring system is used in many common accelerometer designs. A mass is suspended by a pair of springs which displaces when acceleration occurs. An accelerometer contains a mass of 0.080 kg coupled to a spring with spring constant of 4.0 kN m^{-1}. The amplitude of the mass is 20 mm. Calculate:

a) the maximum acceleration

b) the natural frequency of the mass.

Work : What do you understand by spring constant of 25 Nm-1?

It means that to expand a spring by 1 m force needed is 25 N.