CLASSIFIED

International Examinations Papers

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MODULAR MATHEMATICS/CORE-1

TOPIC-Sketching curves

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9. Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$,

(a) express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants.

(3)

(b) Hence factorise $f(x)$ completely.

(2)

(c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes.

(3)

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3. On separate diagrams, sketch the graphs of

(a) $y = (x + 3)^2$,

(3)

(b) $y = (x + 3)^2 + k$, where k is a positive constant.

(2)

Show on each sketch the coordinates of each point at which the graph meets the axes.

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10. (a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

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8. (a) Factorise completely $9x - 4x^3$

(3)

(b) Sketch the curve C with equation

$$y = 9x - 4x^3$$

Show on your sketch the coordinates at which the curve meets the x-axis.

Jn-15

(3)

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

(c) Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found.

(4)

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9. (a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 12$

(3)

(c) Hence find the range of possible values for c.

(4)

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10. (a) On the axes below, sketch the graphs of

(i) $y = x(x+2)(3-x)$

(ii) $y = -\dfrac{2}{x}$

Ja – 11

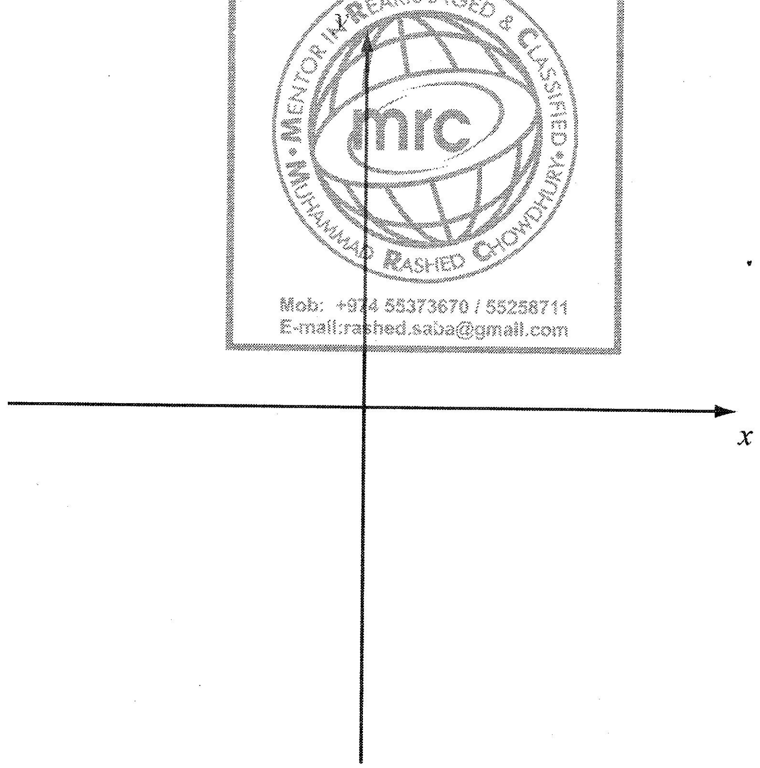
showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0$$

(2)



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10. $$f(x) = x^2 + 4kx + (3 + 11k), \quad \text{where } k \text{ is a constant.}$$

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation f(x) = 0 has no real roots,

(b) find the set of possible values of k.

(4)

Given that $k = 1$,

(c) sketch the graph of y = f(x), showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

Ja - 10

8. The point $P(1, a)$ lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \dfrac{2}{x}$.

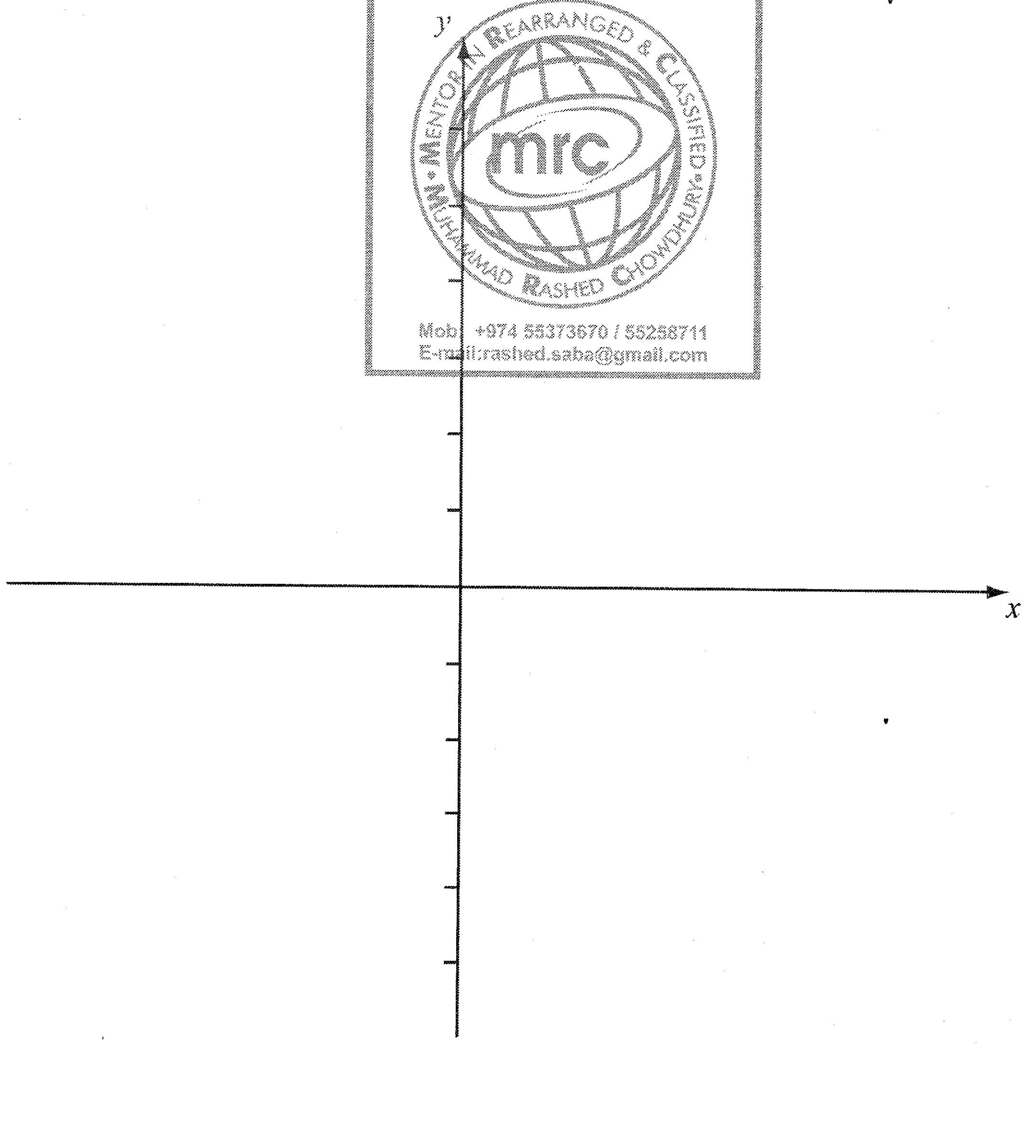
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)



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5.

$$f(x) = x^2 - 8x + 19$$

(a) Express f(x) in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd. Jn-17

(3)

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8. The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$

Ja-12

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

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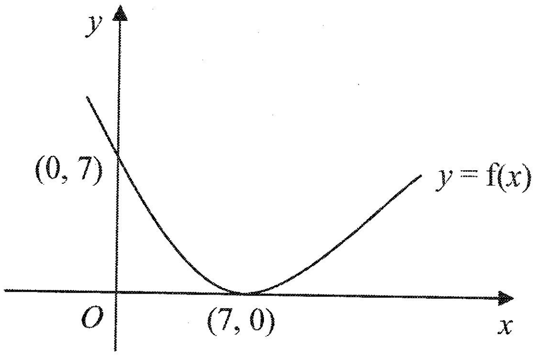

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the point (0, 7) and has a minimum point at (7, 0).

On separate diagrams, sketch the curve with equation

(a) $y = f(x) + 3$,

(3)

(b) $y = f(2x)$.

(2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the y-axis.

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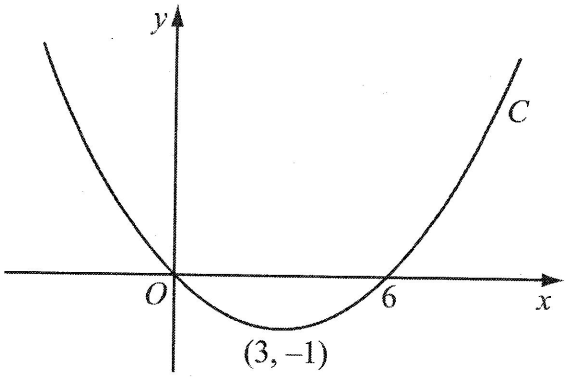

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$,

(3)

(b) $y = -f(x)$,

(3)

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$.

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

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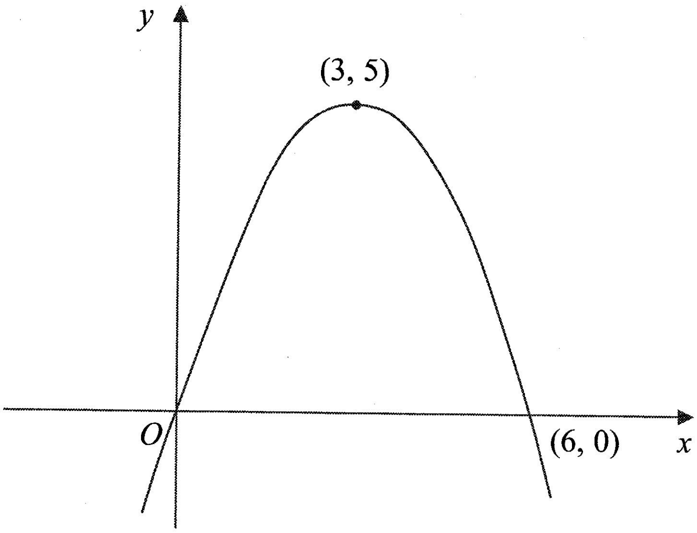
Figure 1



JN - 5

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the origin O and through the point (6, 0). The maximum point on the curve is (3, 5).

On separate diagrams, sketch the curve with equation

(a) $y = 3f(x)$, **(2)**

(b) $y = f(x + 2)$. **(3)**

On each diagram, show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

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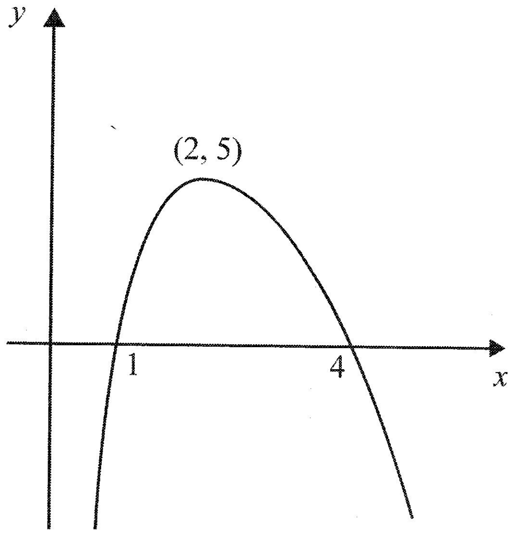

Ja–08

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the points (1, 0) and (4, 0). The maximum point on the curve is (2, 5).
In separate diagrams sketch the curves with the following equations.
On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2\mathrm{f}(x)$,

(3)

(b) $y = \mathrm{f}(-x)$.

(3)

The maximum point on the curve with equation $y = \mathrm{f}(x + a)$ is on the y-axis.

(c) Write down the value of the constant a.

(1)

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6.

Figure 1

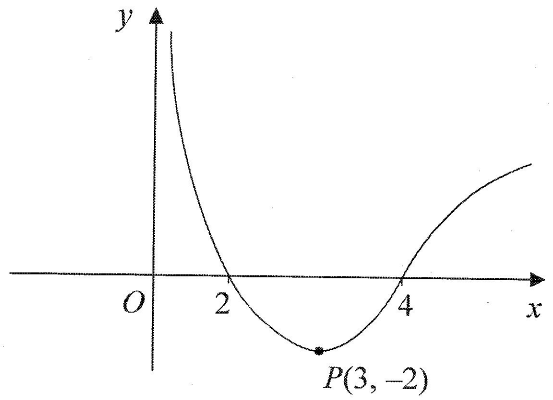

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (2, 0) and (4, 0). The minimum point on the curve is $P(3, -2)$.

In separate diagrams sketch the curve with equation

(a) $y = -f(x)$,

(3)

(b) $y = f(2x)$.

(3)

On each diagram, give the coordinates of the points at which the curve crosses the x-axis, and the coordinates of the image of P under the given transformation.

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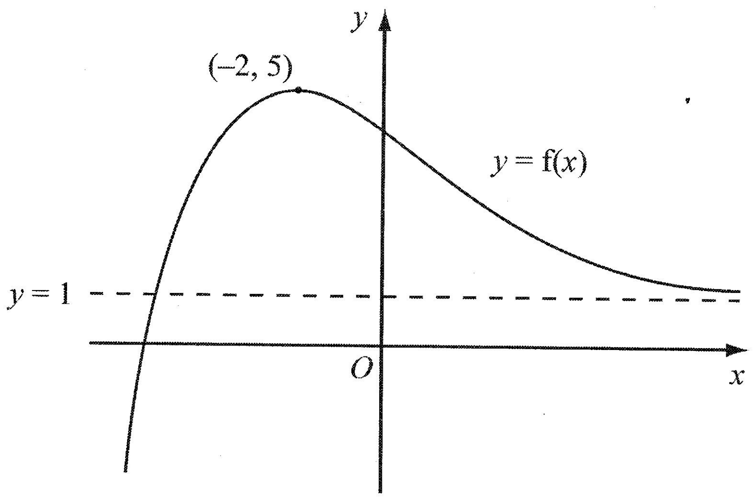

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

The curve has a maximum point $(-2, 5)$ and an asymptote $y = 1$, as shown in Figure 1.

On separate diagrams, sketch the curve with equation

(a) $y = f(x) + 2$ **(2)**

(b) $y = 4f(x)$ **(2)**

(c) $y = f(x + 1)$ **(3)**

On each diagram, show clearly the coordinates of the maximum point and the equation of the asymptote.

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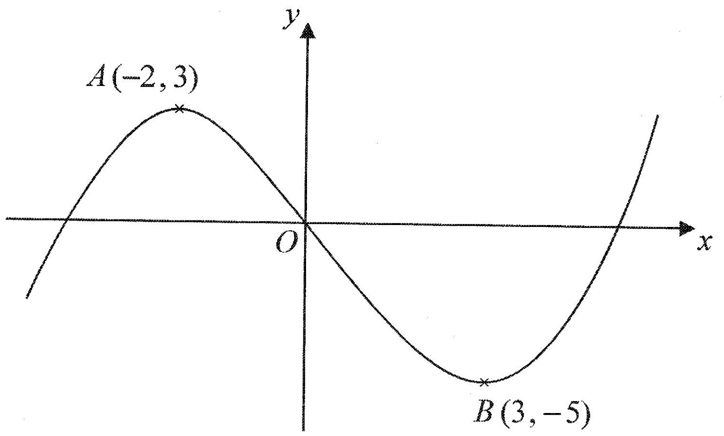

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve has a maximum point A at $(-2, 3)$ and a minimum point B at $(3, -5)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x+3)$

(3)

(b) $y = 2f(x)$

(3)

On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = f(x) + a$ has a minimum at $(3, 0)$, where a is a constant.

(c) Write down the value of a.

(1)

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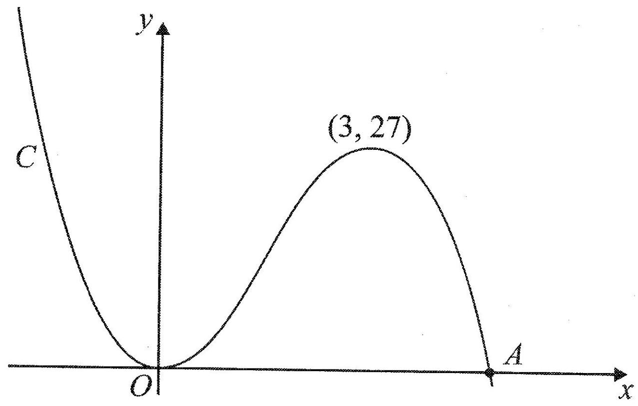

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2(9 - 2x)$$

JN-12

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = \mathrm{f}(x + 3)$

(ii) $y = \mathrm{f}(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = \mathrm{f}(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

(1)

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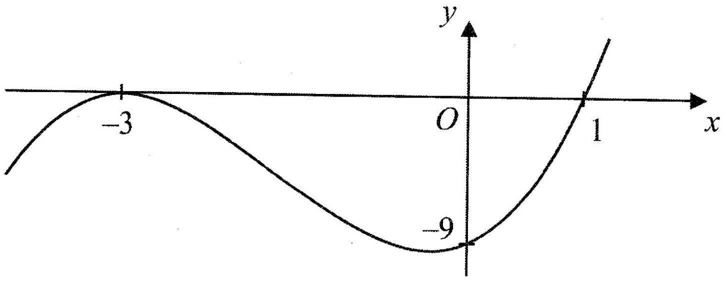

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (x + 3)^2 (x - 1), \qquad x \in \mathbb{R}.$$

The curve crosses the x-axis at (1, 0), touches it at (–3, 0) and crosses the y-axis at (0, –9)

(a) In the space below, sketch the curve C with equation $y = f(x + 2)$ and state the coordinates of the points where the curve C meets the x-axis.

(3)

(b) Write down an equation of the curve C.

(1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C meets the y-axis.

(2)

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JN-16

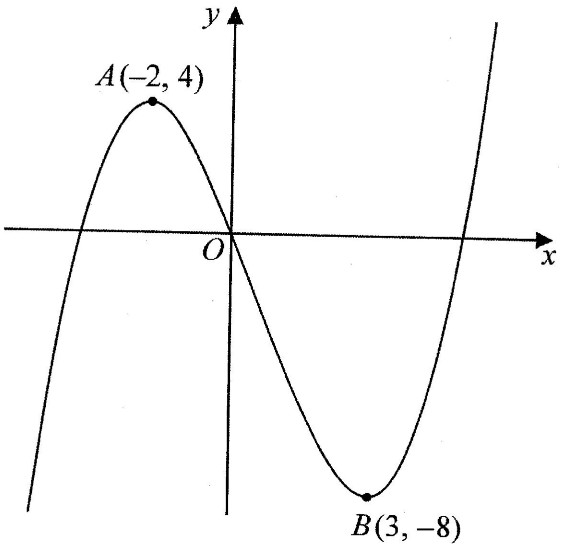


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \text{f}(x)$. The curve has a maximum point A at $(-2, 4)$ and a minimum point B at $(3, -8)$ and passes through the origin O.

On separate diagrams, sketch the curve with equation

(a) $y = 3\text{f}(x)$,

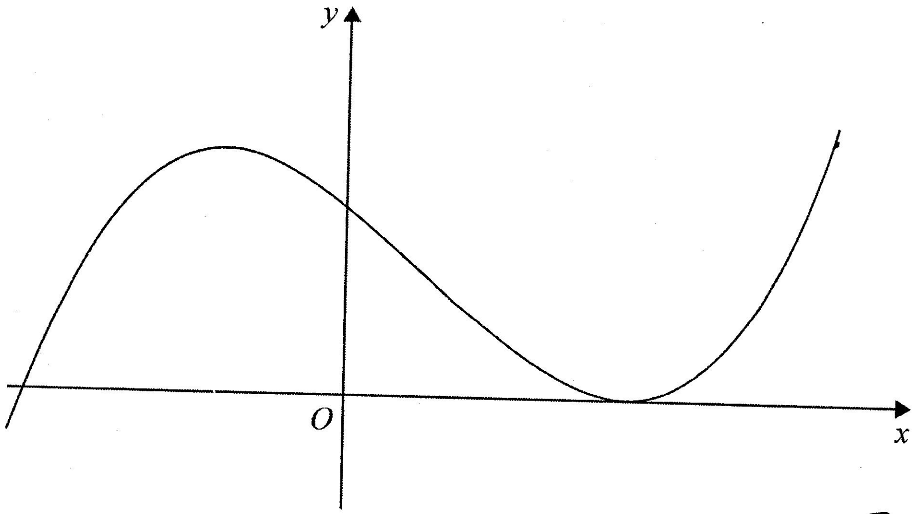
(2)

(b) $y = \text{f}(x) - 4$

(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.

10.



JN-17

Figure 2

Figure 2 shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where

$$f(x) = (2x - 5)^2(x + 3)$$

(a) Given that

(i) the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(b) Show that $f'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = f(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(c) find the x coordinate of point B.

(5)

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Figure 1

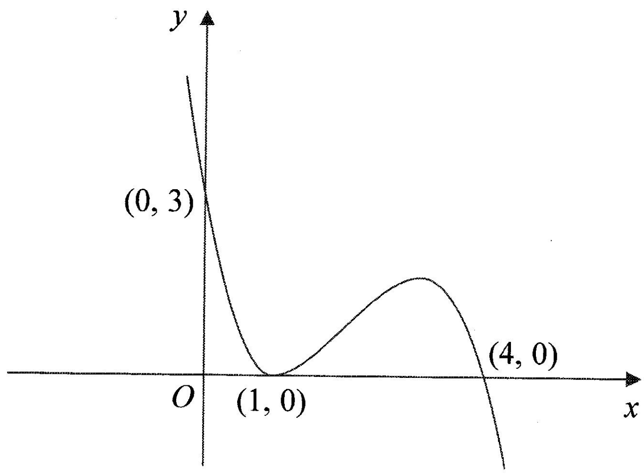

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams sketch the curve with equation

(a) $y = \mathrm{f}(x + 1)$, **(3)**

(b) $y = 2\,\mathrm{f}(x)$, **(3)**

(c) $y = \mathrm{f}\left(\frac{1}{2}x\right)$. **(3)**

On each diagram show clearly the coordinates of all the points where the curve meets the axes.

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Figure 2

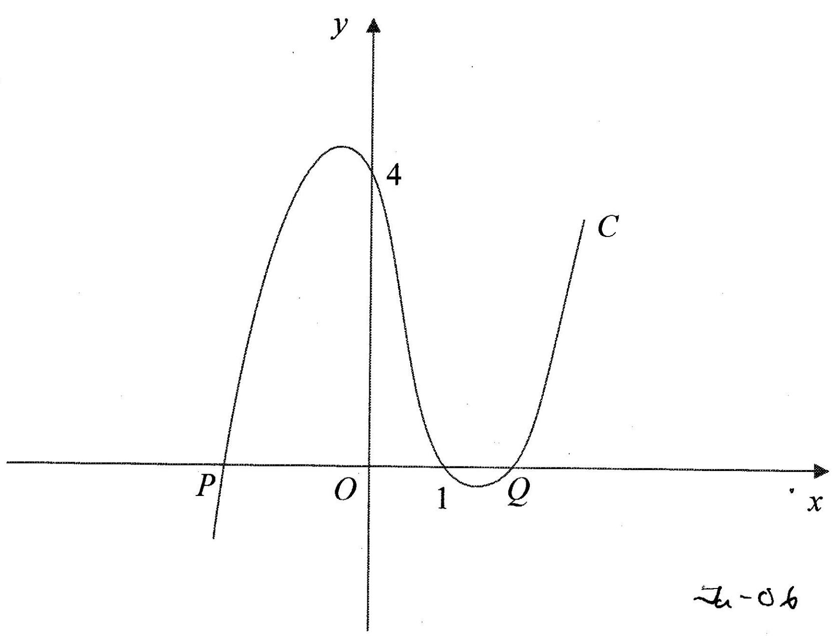

Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, $(1, 0)$ and Q, as shown in Figure 2.

(a) Write down the x-coordinate of P, and the x-coordinate of Q. **(2)**

(b) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 4$. **(3)**

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$. **(2)**

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(d) Find the exact coordinates of R. **(5)**

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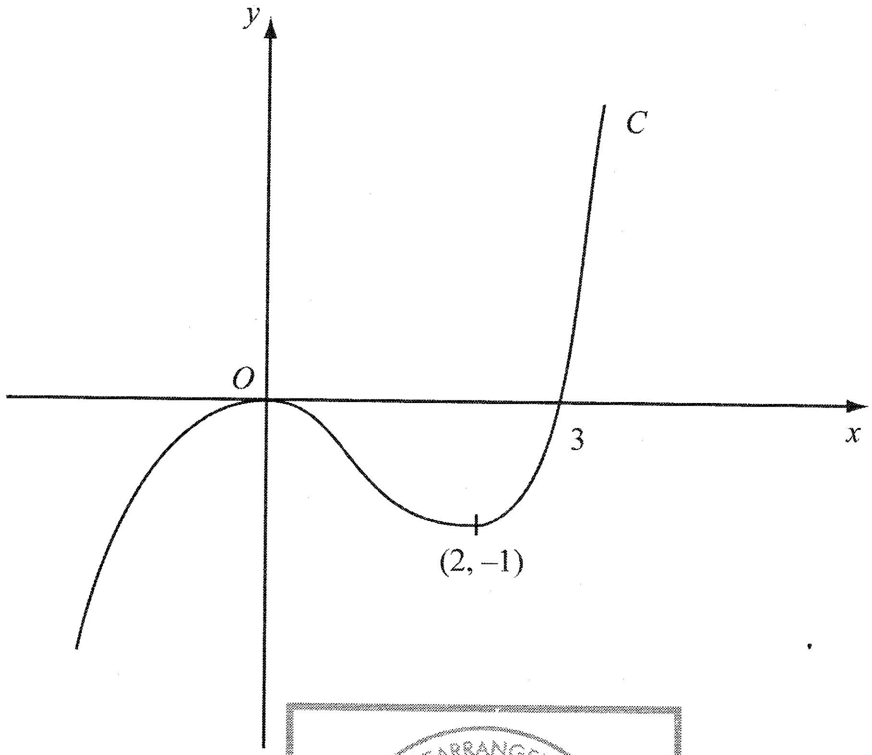

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$. There is a maximum at (0, 0), a minimum at (2, –1) and C passes through (3, 0).

On separate diagrams sketch the curve with equation

(a) $y = \mathrm{f}(x + 3)$,

(3)

(b) $y = \mathrm{f}(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

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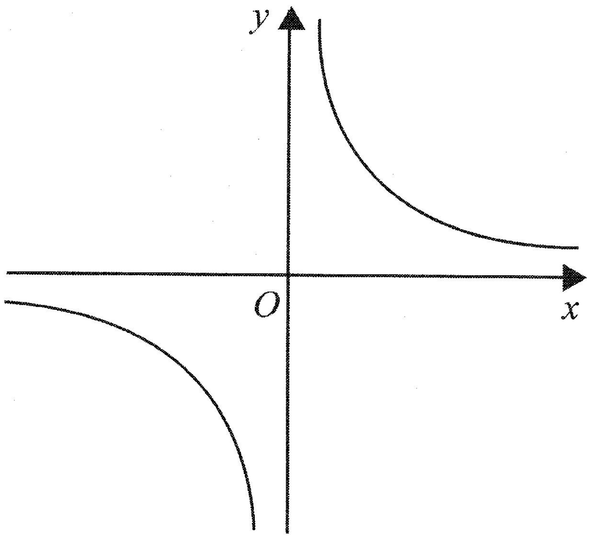

Jn-7

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{3}{x},\ x \neq 0$.

(a) On a separate diagram, sketch the curve with equation $y = \dfrac{3}{x+2},\ x \neq -2$,

showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b) Write down the equations of the asymptotes of the curve in part (a).

(2)

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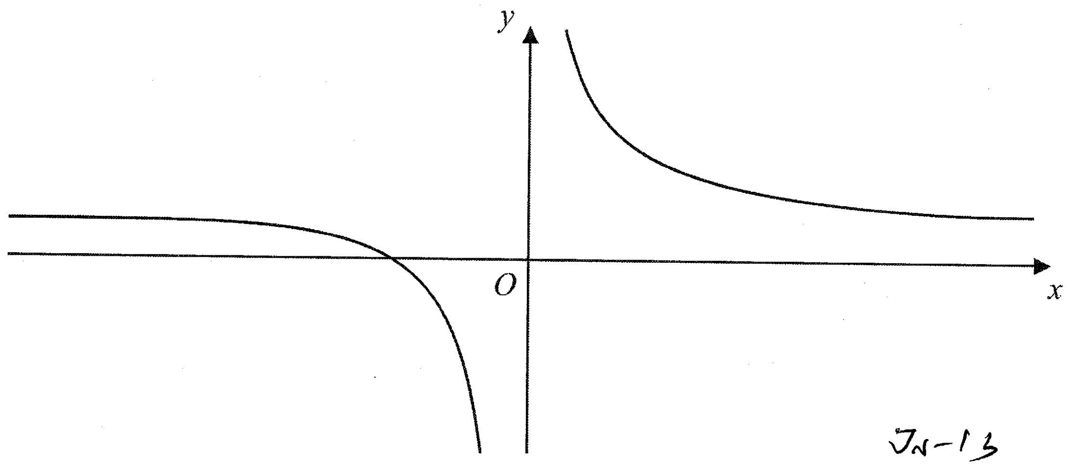

Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4, \ x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.

(1)

(b) Give the equations of the asymptotes to H.

(2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$.

(5)

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd.

(3)

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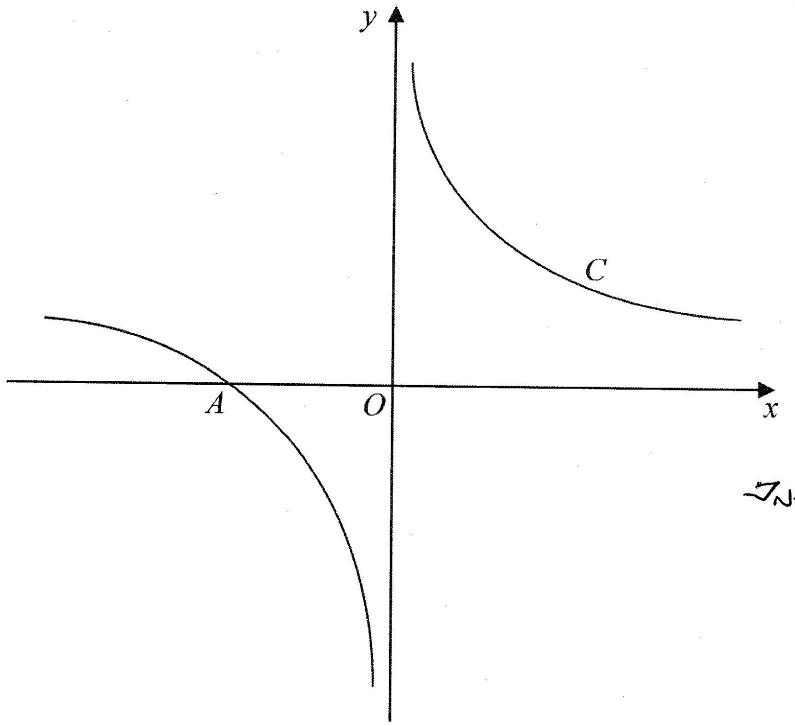

JN-14

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{x} + 1, \qquad x \neq 0$$

The curve C crosses the x-axis at the point A.

(a) State the x coordinate of the point A.

(1)

The curve D has equation $y = x^2(x - 2)$, for all real values of x.

(b) A copy of Figure 1 is shown on page 7.
On this copy, sketch a graph of curve D.
Show on the sketch the coordinates of each point where the curve D crosses the coordinate axes.

(3)

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x - 2) = \frac{1}{x} + 1$$

(1)

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Question 4 continued

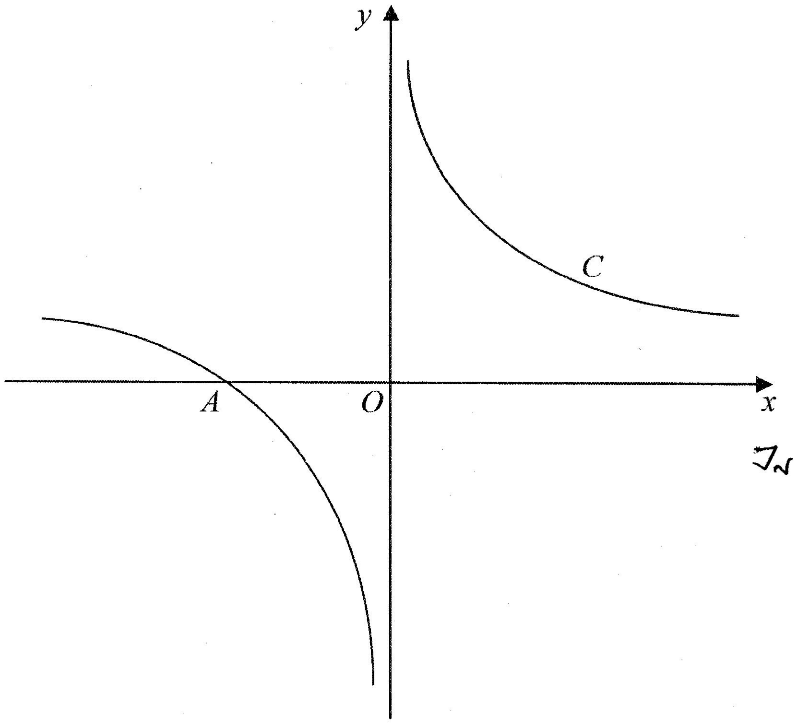

Figure 1

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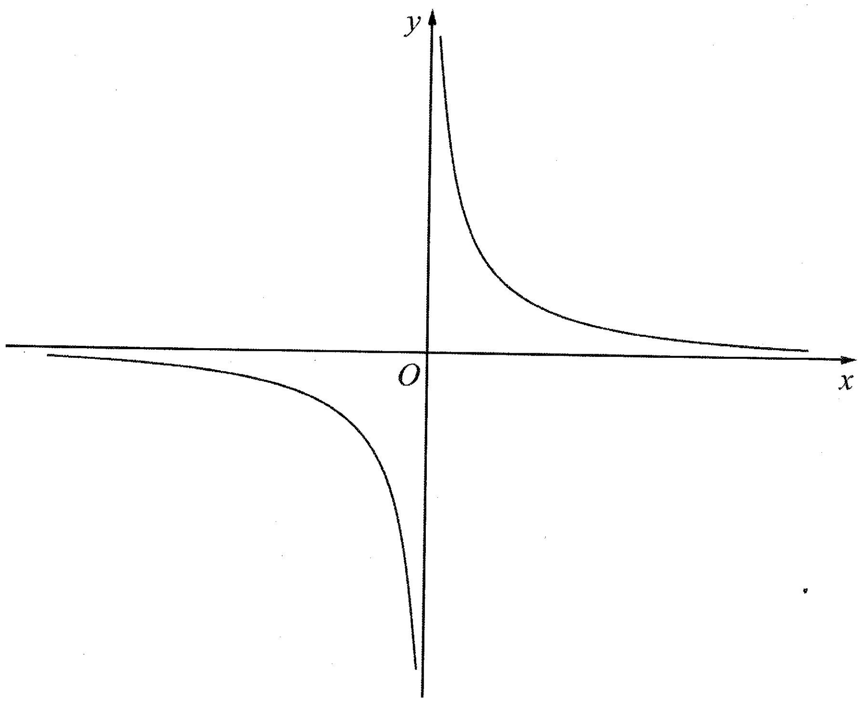

Figure 1

Ja-13

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}$, $x \neq 0$

The curve C has equation $y = \frac{2}{x} - 5$, $x \neq 0$, and the line l has equation $y = 4x + 2$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

(5)

(b) Write down the equations of the asymptotes of the curve C.

(2)

(c) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$

(5)

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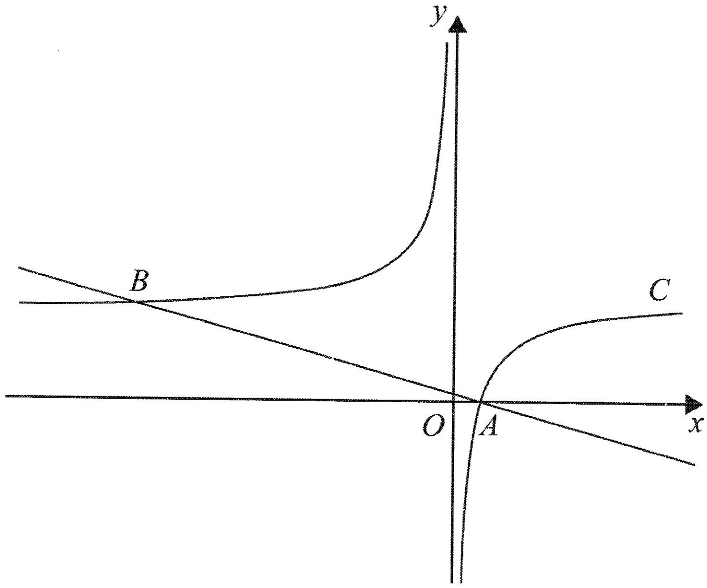

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \quad x \neq 0$$

The curve crosses the x-axis at the point A.

(a) Find the coordinates of A.
(1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0$$

(6)

The normal to C at A meets C again at the point B, as shown in Figure 2.

(c) Find the coordinates of B.
(4)

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Ja-11

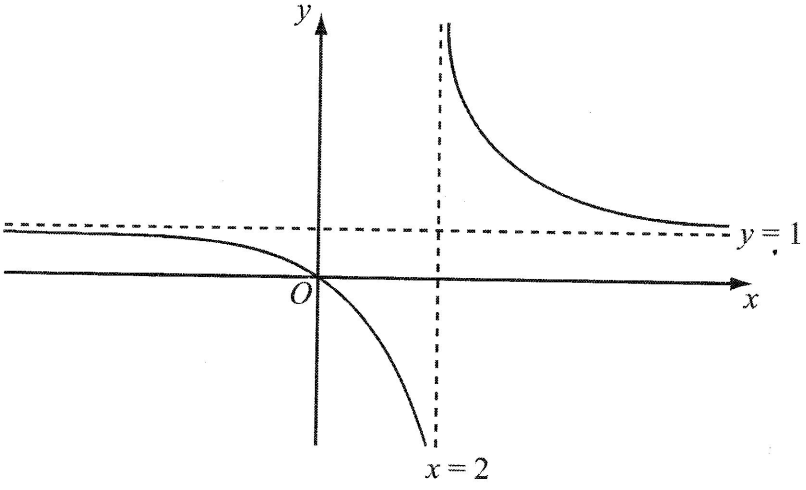

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \quad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x-1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x-1)$ crosses the coordinate axes.

(4)