www.mrc-papers.com

CLASSIFIED

International Examinations Papers

Mob: +974 55249797 / 55258711
E-mail:rashed.saba@gmail.com

Pure Mathematics-1

TOPIC- Trigonometry

TRIGONOMETRY-Identity

1 Prove the identity $\dfrac{\sin x}{1 - \sin x} - \dfrac{\sin x}{1 + \sin x} \equiv 2\tan^2 x$. [3]

2 Prove the identity

$$\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} \equiv \frac{2}{\cos x}.$$ [4]

TRIGONOMETRY-Identity

3 **(i)** Show that the equation $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$ can be written as a quadratic equation in $\tan\theta$. [2]

(ii) Hence, or otherwise, solve the equation in part **(i)** for $0° \leqslant \theta \leqslant 180°$. [3]

4 **(i)** Prove the identity $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 \equiv \dfrac{1-\cos x}{1+\cos x}$. [4]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leqslant x \leqslant 2\pi$. [3]

TRIGONOMETRY-Identity

J-14-12
5

5 **(i)** Show that the equation $1 + \sin x \tan x = 5 \cos x$ can be expressed as

$$6\cos^2 x - \cos x - 1 = 0.$$ [3]

(ii) Hence solve the equation $1 + \sin x \tan x = 5 \cos x$ for $0° \leqslant x \leqslant 180°$.

6 **(i)** Show that $\cos^4 x \equiv 1 - 2\sin^2 x + \sin^4 x$. [1]

J-16-11
7

(ii) Hence, or otherwise, solve the equation $8\sin^4 x + \cos^4 x = 2\cos^2 x$ for $0° \leqslant x \leqslant 360°$. [5]

TRIGONOMETRY-Identity

S-16-12-7

7 (i) Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta}-\dfrac{1-\cos\theta}{1+\cos\theta}\equiv\dfrac{4}{\sin\theta\tan\theta}$. [4]

S-11-13

8

8 (i) Prove the identity $\left(\dfrac{1}{\sin\theta}-\dfrac{1}{\tan\theta}\right)^2\equiv\dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin\theta}-\dfrac{1}{\tan\theta}\right)^2=\dfrac{2}{5}$, for $0°\leqslant\theta\leqslant360°$. [4]

S-14-11 9

9 **(i)** Prove the identity $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} \equiv \dfrac{1}{\tan\theta}$. [4]

(ii) Hence solve the equation $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} = 4\tan\theta$ for $0° < \theta < 180°$. [3]

S-12-13 1

10 **(i)** Prove the identity $\tan^2\theta - \sin^2\theta \equiv \tan^2\theta\sin^2\theta$. [3]

(ii) Use this result to explain why $\tan\theta > \sin\theta$ for $0° < \theta < 90°$. [

TRIGONOMETRY-Identity

s7-10-12 1

11 (i) Show that the equation

$$3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

(ii) Solve the equation $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$, for $0° \leqslant x \leqslant 360°$. [2]

7-10-12 2

12 Prove the identity

$$\tan^2 x - \sin^2 x \equiv \tan^2 x \sin^2 x.$$

[4]

TRIGONOMETRY-Identity

13 (i) Show that the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$ can be expressed as $\tan\theta = 3$. [2]

(ii) Hence solve the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$, for $0° \leqslant \theta \leqslant 360°$. [2]

14 (i) Prove the identity $\dfrac{\sin x \tan x}{1 - \cos x} \equiv 1 + \dfrac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\dfrac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0° \leqslant x \leqslant 360°$. [3]

TRIGONOMETRY-Identity

5 (i) Prove the identity $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1+\sin\theta} \equiv \tan\theta$. [4]

(ii) Solve the equation $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1+\sin\theta} + 2 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

6 (i) Show that the equation $2\cos x = 3\tan x$ can be written as a quadratic equation in $\sin x$. [3]

(ii) Solve the equation $2\cos 2y = 3\tan 2y$, for $0° \leqslant y \leqslant 180°$. [4]

TRIGONOMETRY-Identity

5-11-12
5

17 (i) Prove the identity $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} \equiv 1 + \dfrac{1}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} = 4$, for $0° \leqslant \theta \leqslant 360°$.

7-14-13
5

18 (i) Show that $\sin^4\theta - \cos^4\theta \equiv 2\sin^2\theta - 1$. [3]

(ii) Hence solve the equation $\sin^4\theta - \cos^4\theta = \frac{1}{2}$ for $0° \leqslant \theta \leqslant 360°$. [4]

19 **(i)** Prove the identity $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$. [3]

(ii) Hence solve the equation $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = 3\sin x - 2\cos x$ for $0 \leqslant x \leqslant 2\pi$. [3]

20 **(i)** Show that the equation $2\tan^2\theta\cos\theta = 3$ can be written in the form $2\cos^2\theta + 3\cos\theta - 2 = 0$. [2]

(ii) Hence solve the equation $2\tan^2\theta\cos\theta = 3$, for $0° \leqslant \theta \leqslant 360°$. [3]

21 (i) Show that the equation $4\sin^4\theta + 5 = 7\cos^2\theta$ may be written in the form $4x^2 + 7x - 2 = 0$, where $x = \sin^2\theta$. [1]

(ii) Hence solve the equation $4\sin^4\theta + 5 = 7\cos^2\theta$, for $0° \leqslant \theta \leqslant 360°$. [4]

22 (i) Show that the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as

$$4\sin^2\theta - 15\sin\theta - 4 = 0.$$

[3]

(ii) Hence solve the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

TRIGONOMETRY-Solving

23 **(i)** Show that $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{1}{\sin^2\theta-\cos^2\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}=3$, for $0^\circ\leqslant\theta\leqslant360^\circ$. [4]

24 **(i)** Show that the equation $2\sin x\tan x+3=0$ can be expressed as $2\cos^2x-3\cos x-2=0$. [2]

(ii) Solve the equation $2\sin x\tan x+3=0$ for $0^\circ\leqslant x\leqslant360^\circ$. [3]

25 (i) Prove the identity $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \equiv \dfrac{\tan\theta - 1}{\tan\theta + 1}$.

(ii) Hence solve the equation $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{6}$, for $0° \leqslant \theta \leqslant 180°$.

26 (i) Show that the equation $2\tan^2\theta\sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta\sin^2\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

TRIGONOMETRY-Solving

8-13-12
5

27 It is given that $\mathbf{a} = \sin\theta - 3\cos\theta$ and $\mathbf{b} = 3\sin\theta + \cos\theta$, where $0° \leqslant \theta \leqslant 360°$.

(i) Show that $\mathbf{a}^2 + \mathbf{b}^2$ has a constant value for all values of θ. [3]

(ii) Find the values of θ for which $2\mathbf{a} = \mathbf{b}$. [4]

7-9-12
5

28 (i) Prove the identity $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$. [3]

(ii) Solve the equation $(\sin x + \cos x)(1 - \sin x \cos x) = 9\sin^3 x$ for $0° \leqslant x \leqslant 360°$. [3]

TRIGONOMETRY-Solving

29 **(i)** Show that the equation $3\sin x\tan x = 8$ can be written as $3\cos^2 x + 8\cos x - 3 = 0$. [3]

(ii) Hence solve the equation $3\sin x\tan x = 8$ for $0° \leqslant x \leqslant 360°$. [3]

30 **(i)** Prove the identity $\tan x + \dfrac{1}{\tan x} \equiv \dfrac{1}{\sin x\cos x}$. [2]

(ii) Solve the equation $\dfrac{2}{\sin x\cos x} = 1 + 3\tan x$, for $0° \leqslant x \leqslant 180°$. [4]

TRIGONOMETRY-Solving

3 (i) Given that

$$3\sin^2 x - 8\cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$ [3]

(ii) Hence solve the equation

$$3\sin^2(\theta + 70°) - 8\cos(\theta + 70°) - 7 = 0$$

for $0° \leqslant \theta \leqslant 180°$. [4]

32 **(i)** Prove the identity $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} \equiv \dfrac{2}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{3}{\cos\theta}$ for $0° \leqslant \theta \leqslant 360°$. [3]

33 **(i)** Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \frac{1-\sin\theta}{1+\sin\theta}$. [3]

....

....

....

....

....

....

....

....

....

....

....

......

S-17-12-7

(ii) Hence solve the equation $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$, for $0° \leqslant \theta \leqslant 360°$. [3]

....

....

....

....

....

....

....

....

....

....

S 17-12-7

34 Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

35 Find all the values of x in the interval $0° \leqslant x \leqslant 180°$ which satisfy the equation $\sin 3x + 2\cos 3x = 0$. [4]

TRIGONOMETRY-Solving

36 Solve the equation $\dfrac{13\sin^2\theta}{2+\cos\theta} + \cos\theta = 2$ for $0° \leqslant \theta \leqslant 180°$. [4]

37 Solve the equation $\sin 2x = 2\cos 2x$, for $0° \leqslant x \leqslant 180°$. [4]

J-12-13 3

38 Solve the equation $7\cos x + 5 = 2\sin^2 x$, for $0° \leqslant x \leqslant 360°$. [4]

N-10-13 3

39 Solve the equation $15\sin^2 x = 13 + \cos x$ for $0° \leqslant x \leqslant 180°$. [4]

S7-12-13 9

40 **(i)** Solve the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 360°$. [5]

(ii) How many solutions has the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 1080°$? [1]

12

S7-16-13 8(ii)

41 **(ii)** Find the solutions to the equation $3\sin 2x \tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]