INTEGRATION-11

1.

Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

JU-14-4

2.

Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2} \right) dx$.

(5)

JU-6-2

3.

Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \mathrm{d}x$.

JU-6-2

4.

$$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,\mathrm{d}x$.

(4)

JA-7-1

INTEGRATION-11

5.

Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

JU-7-1

6.

Use calculus to find the value of

$$\int_1^4 \left(2x + 3\sqrt{x}\right) dx\,.$$

(5)

JU-9-1

7.

$$y = 3^x + 2x$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\,dx$.

(4)

JU-10-1

8.

The speed, v m s^{-1}, of a train at time t seconds is given by

$$v = \sqrt{(1.2^t - 1)}, \qquad 0 \le t \le 30.$$

The following table shows the speed of the train at 5 second intervals.

t	0	5	10	15	20	25	30
v	0	1.22	2.28		6.11		

(a) Complete the table, giving the values of v to 2 decimal places.

(3)

The distance, s metres, travelled by the train in 30 seconds is given by

$$s = \int_0^{30} \sqrt{(1.2^t - 1)} \; dt .$$

(b) Use the trapezium rule, with all the values from your table, to estimate the value of s.

(3)

JA-6-6

9.

$$y = \sqrt{(10x - x^2)}.$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\, dx$.

(4)

JA-9-3

INTEGRATION-11

10.

(a) In the space provided, sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

(b) Complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(c) Use the trapezium rule, with all the values from your table, to find an approximation for the value of $\int_0^1 3^x \, dx$.

(4)

JU-6-5

INTEGRATION-11

11.

$$y = \sqrt{(5^x + 2)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.5	1	1.5	2
y			2.646	3.630	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)}\,dx$.

(4)

JU-8-2

5.

12.

(a) $y = 5^x + \log_2(x + 1), \quad 0 \leqslant x \leqslant 2$

Complete the table below, by giving the value of y when $x = 1$

x	0	0.5	1	1.5	2
y	1	2.821		12.502	26.585

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^2 (5^x + \log_2(x + 1))\,dx$$

giving your answer to 2 decimal places.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^2 (5 + 5^x + \log_2(x + 1))\,dx$$

giving your answer to 2 decimal places.

(1)

JA-17-3

13.

(*a*) Sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

(*b*) Copy and complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(*c*) Use the trapezium rule, with all the values from your tables, to find an approximation for the value of $\int_0^1 3^x \ dx$.

(4)

JU-6-5

14. 12.

A river, running between parallel banks, is 20 m wide. The depth, y metres, of the river measured at a point x metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20 - x)}, \quad 0 \leqslant x \leqslant 20.$$

(a) Complete the table below, giving values of y to 3 decimal places.

x	0	4	8	12	16	20
y	0		2.771			0

(2)

(b) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

(4)

Given that the cross-sectional area is constant and that the river is flowing uniformly at $2\ \text{ms}^{-1}$,

(c) estimate, in m^3, the volume of water flowing per minute, giving your answer to 3 significant figures.

(2)

JU-5-6

15.

The curve C has equation

$$y = x\sqrt{(x^3 + 1)}, \qquad 0 \leqslant x \leqslant 2.$$

(a) Complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530			6

(2)

(b) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\,dx$, giving your answer to 3 significant figures.

(4)

$$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of $\int_0^1 \sqrt{(3^x + x)}\,dx$

You must show clearly how you obtained your answer.

(4)

JU-12-7

INTEGRATION-11

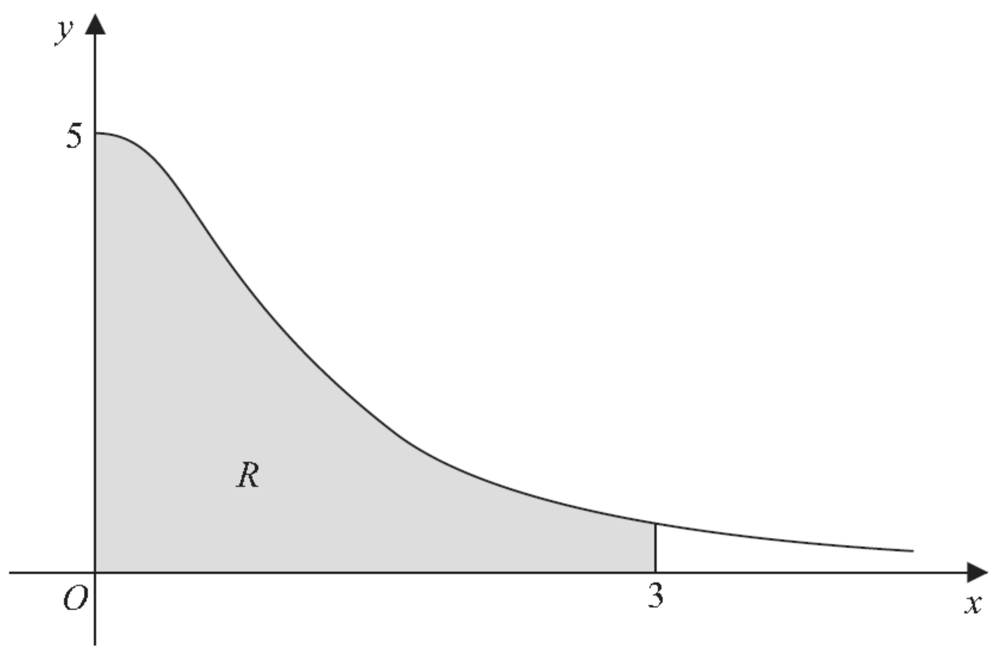
16.

$$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)



INTEGRATION-11

Figure 1 shows the region R which is bounded by the curve with equation $y = \dfrac{5}{(x^2+1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2+1)}\right) dx$$

giving your answer to 2 decimal places.

(2)

JU-13-4

INTEGRATION-11

17.

$$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

(4)

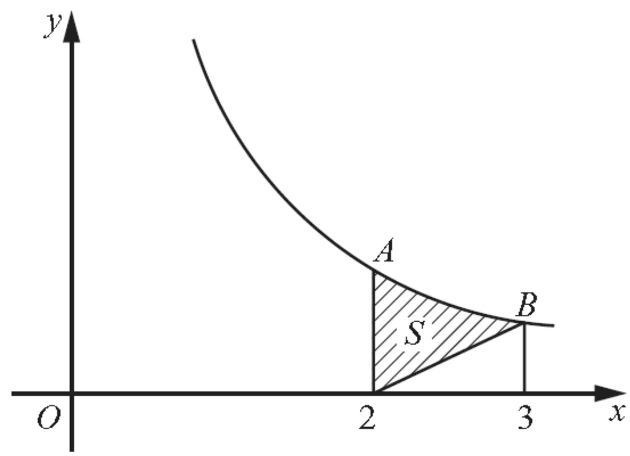


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

JA-11-6

INTEGRATION-11

18.

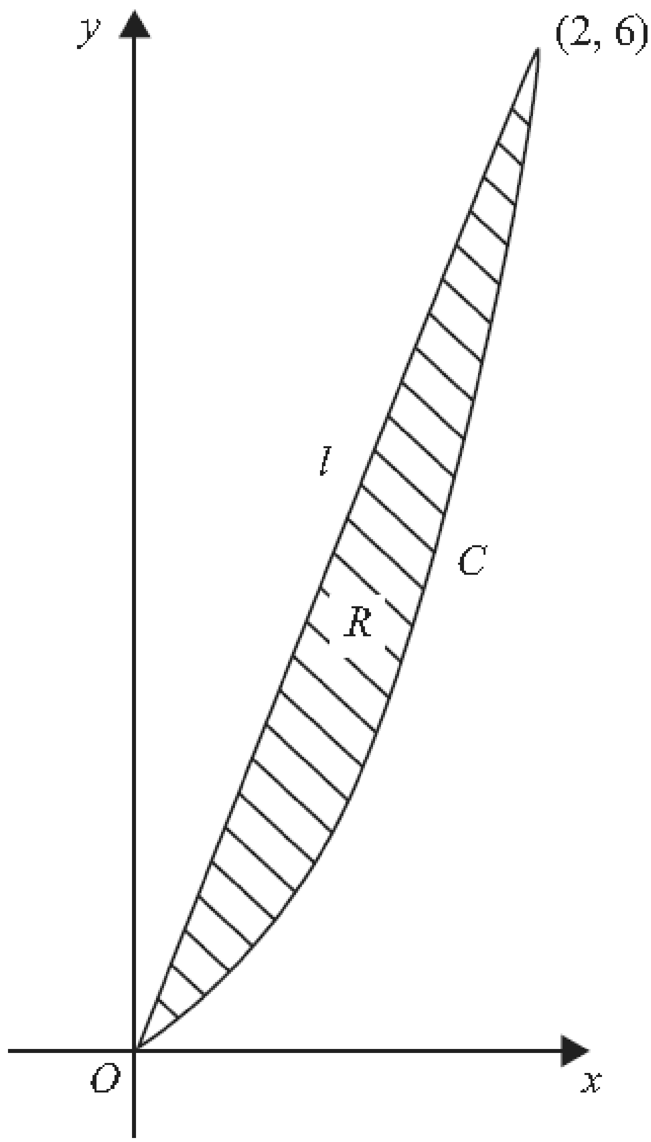


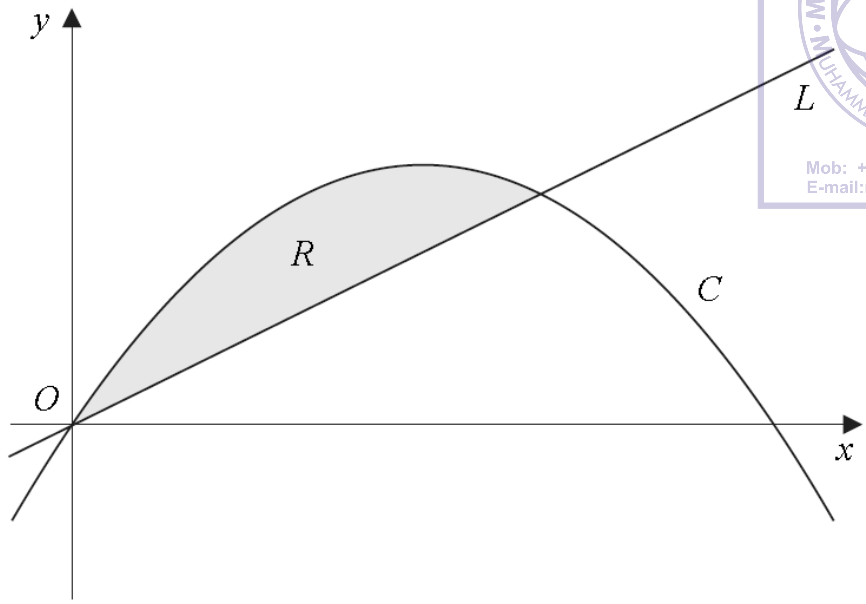
Figure 2 shows the curve C with equation $y = x\sqrt{(x^3+1)}$, $0 \leqslant x \leqslant 2$, and the straight line segment l, which joins the origin and the point (2, 6). The finite region R is bounded by C and l.

(c) Use your answer to part (b) to find an approximation for the area of R, giving your answer to 3 significant figures.

(3)

JU-7-5

19.



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

20.

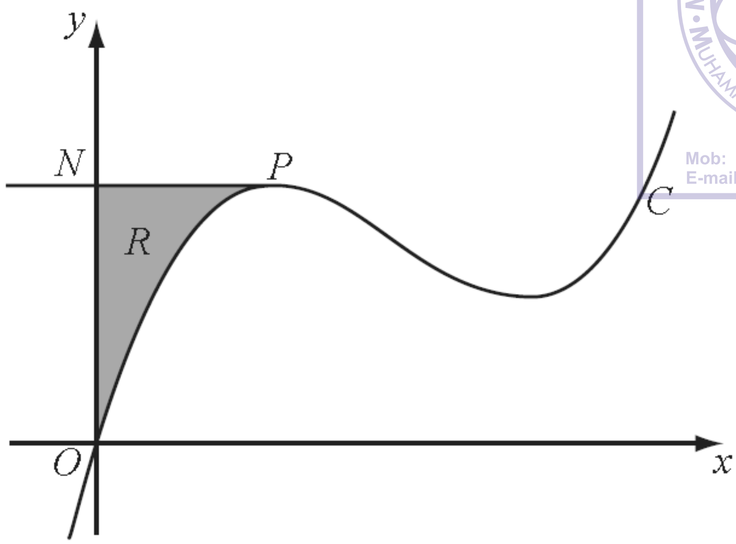


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

JU-10-8

21.

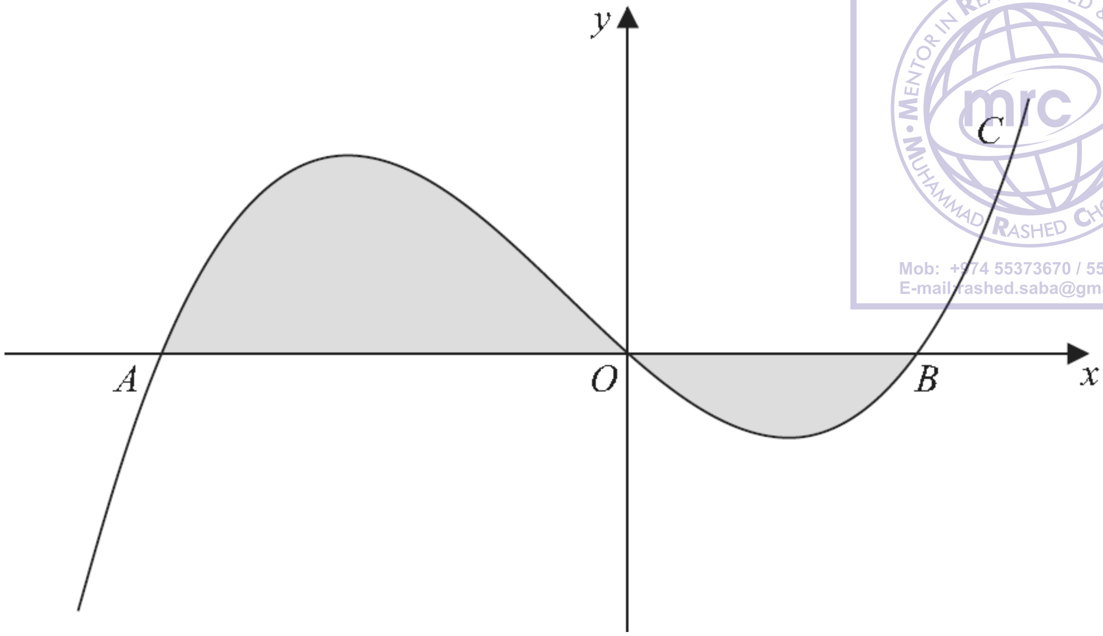


Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

JU-13-6

22.

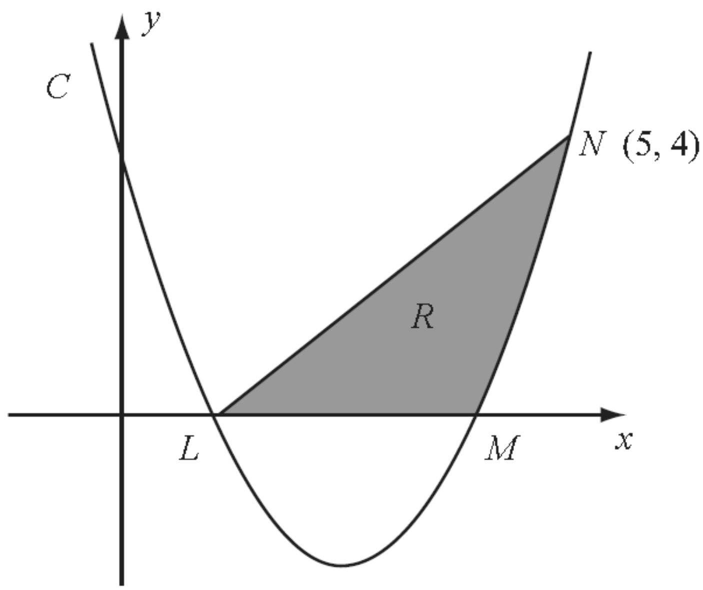


Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M.

(2)

(b) Show that the point N (5, 4) lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4)\, dx$.

(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)

JA-10-7

INTEGRATION-11

23.

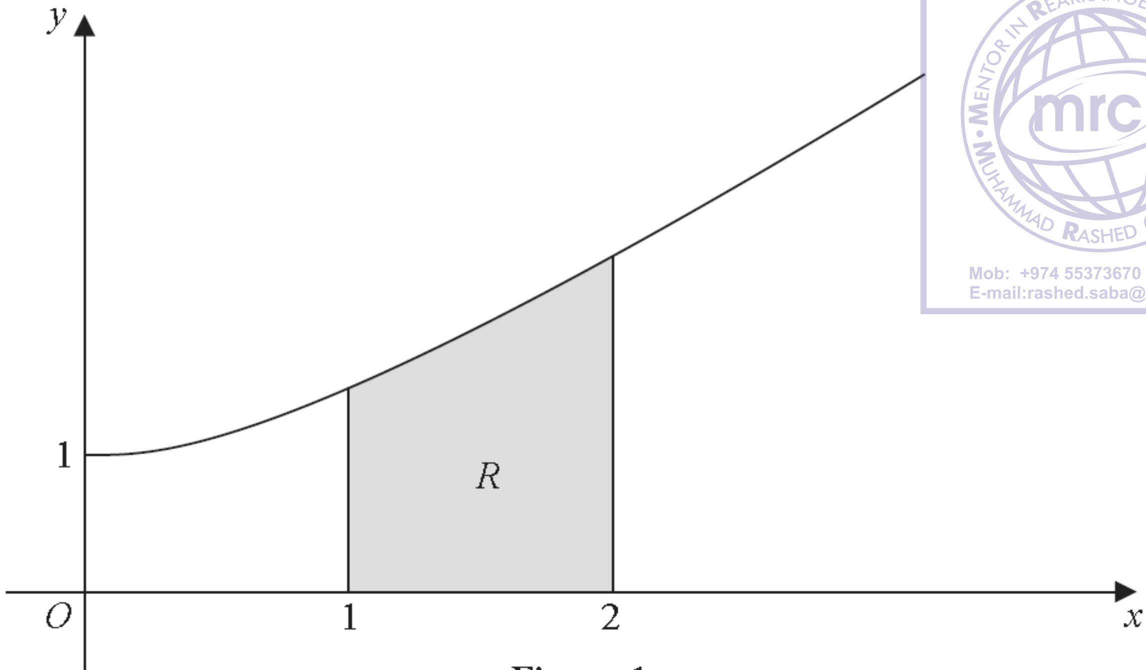


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414		1.803	2.016	2.236

(a) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

JU-14-1

24.

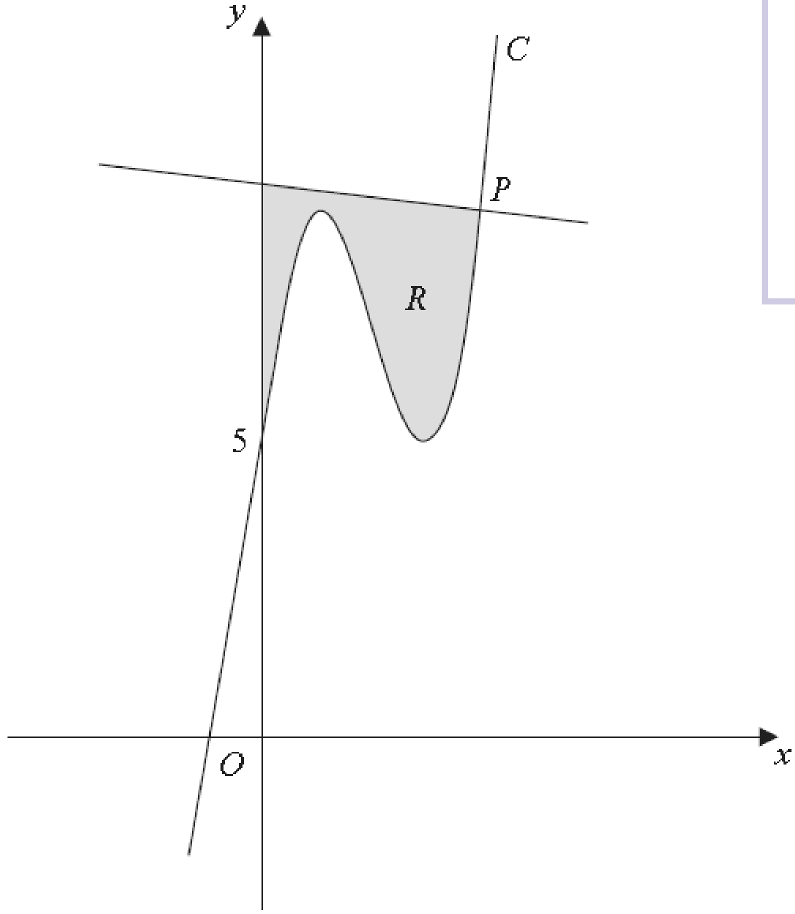


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = x^3 - 6x^2 + 9x + 5$$

The point $P(4, 9)$ lies on C.

(a) Show that the normal to C at the point P has equation

$$x + 9y = 85$$

(6)

INTEGRATION-11

The region R, shown shaded in Figure 1, is bounded by the curve C, the y-axis and the normal to C at P.

(b) Showing all your working, calculate the exact area of R. **(7)**

JA-14-7

25.

Figure 3

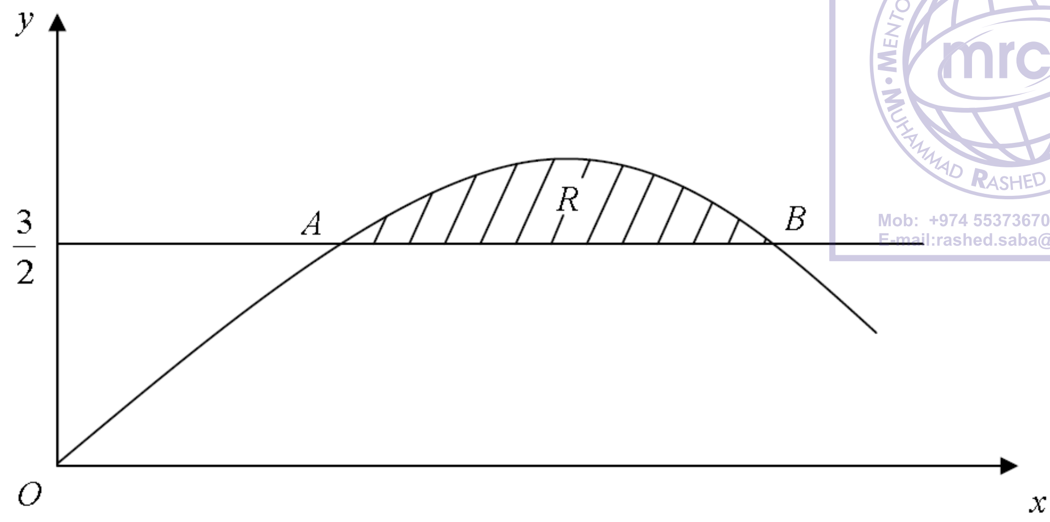


Figure 3 shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve.

Find

(*a*) the x-coordinates of the points A and B,

(4)

(*b*) the exact area of R.

(6)

JA-6-9

26.

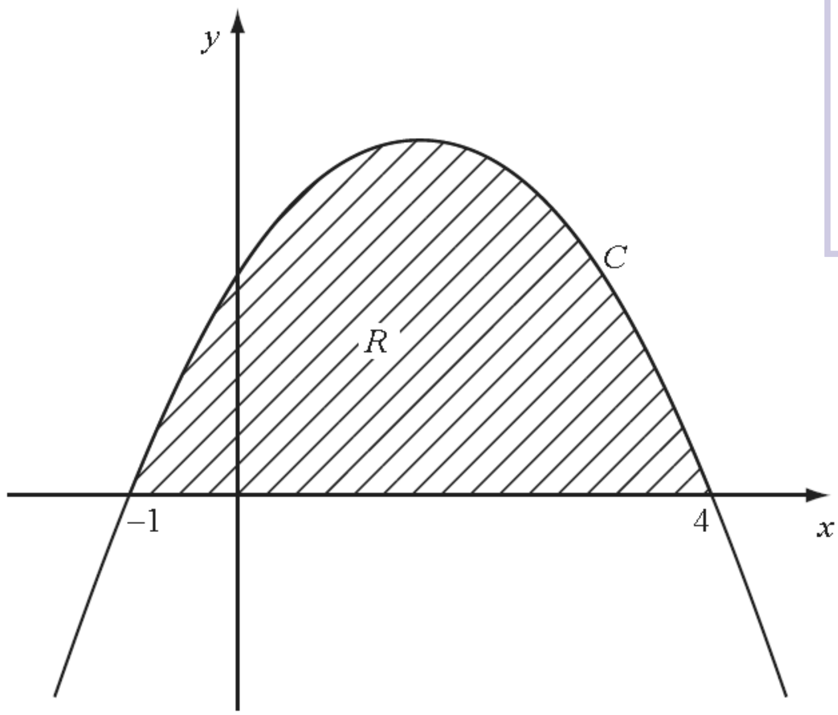


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

JA-9-2

27.

Figure 3

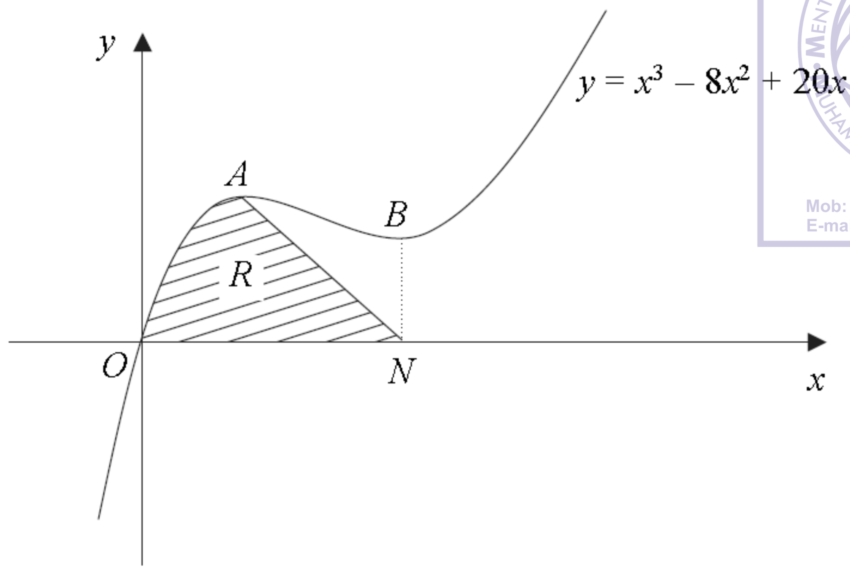


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$.

(3)

JU-6-10

28.

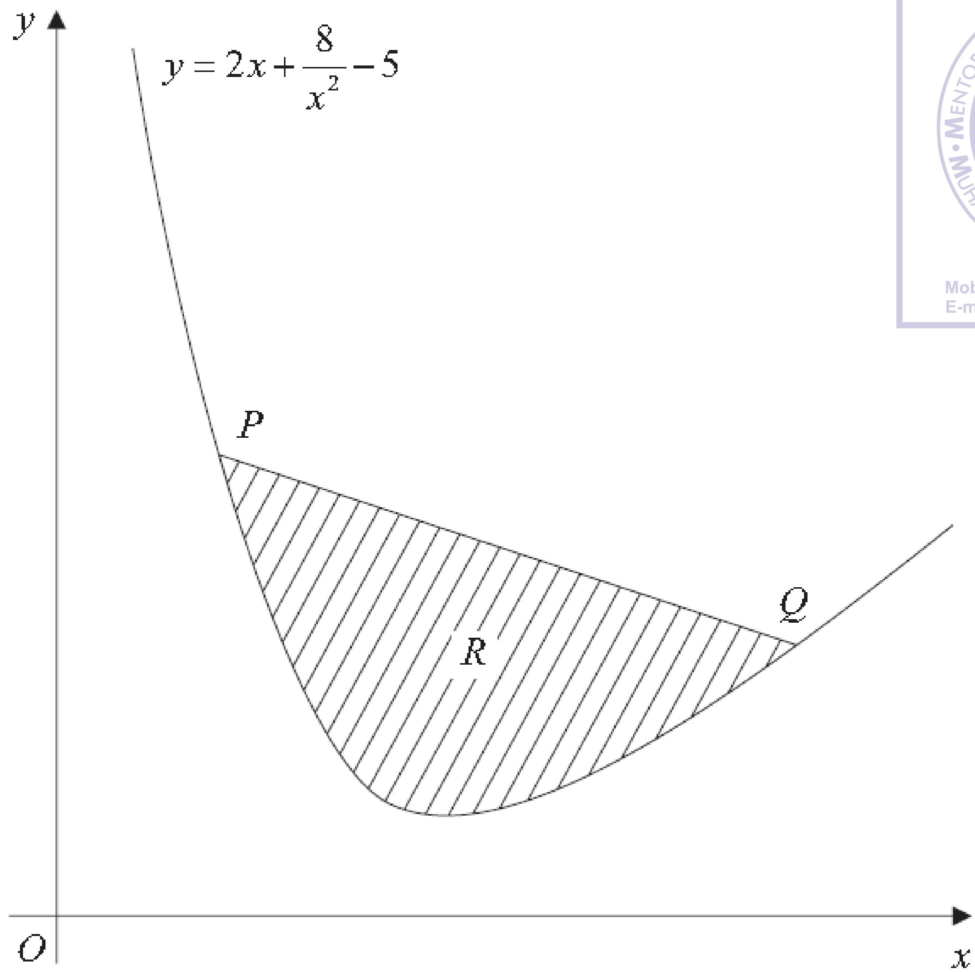


Figure 1 shows part of the curve C with equation $y = 2x + \frac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in Figure 1, is bounded by C and the straight line joining P and Q.

(a) Find the exact area of R.

(8)

(b) Use calculus to show that y is increasing for $x > 2$.

(4)

JU-5-10

29.

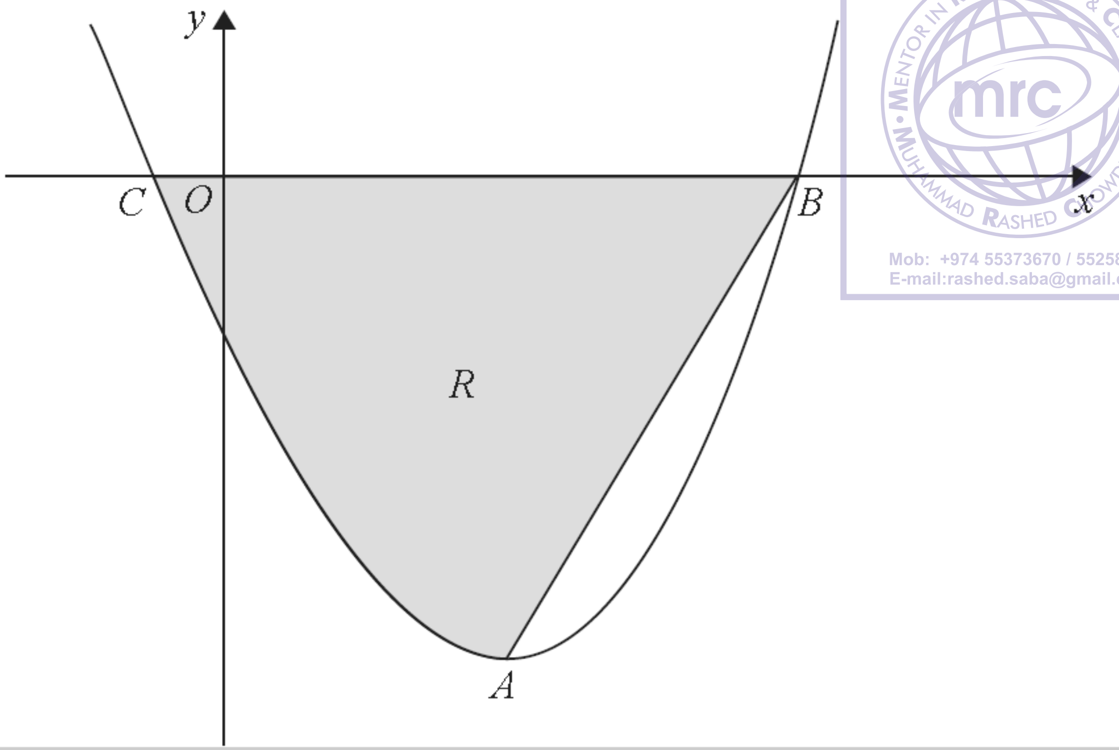


Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

JA-17-10

30.

The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leqslant x \leqslant 4$$

(a) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 \left(8 - 2^{x-1}\right) \mathrm{d}x$

(3)

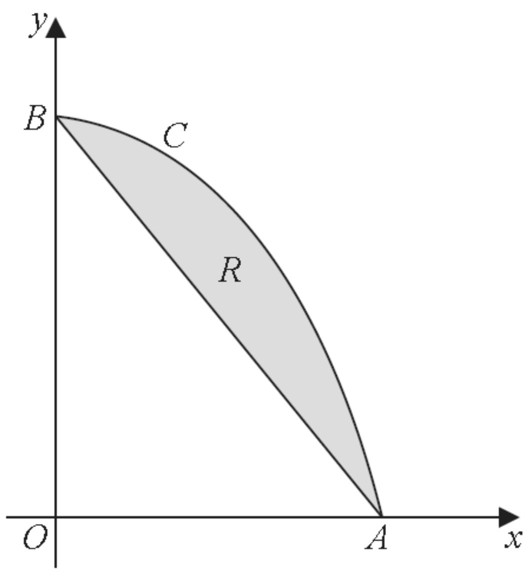


Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}, \quad 0 \leqslant x \leqslant 4$

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

JU-16-2

31.

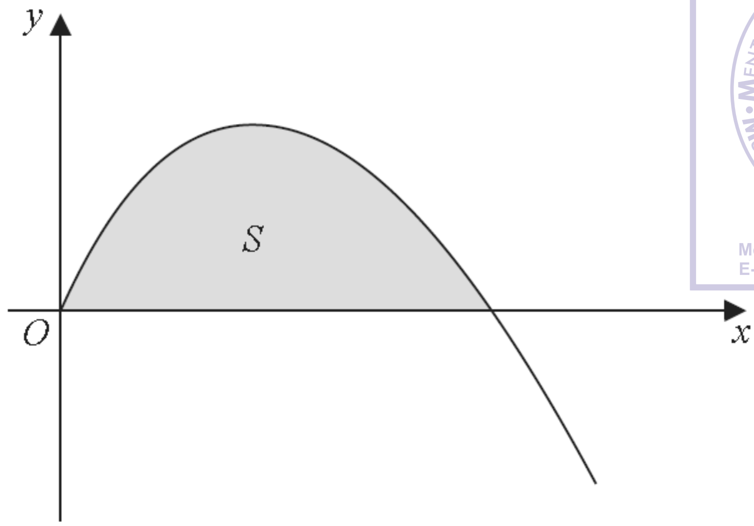


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x$$

(3)

(b) Hence find the area of S.

(3)

JU-16-7

32.

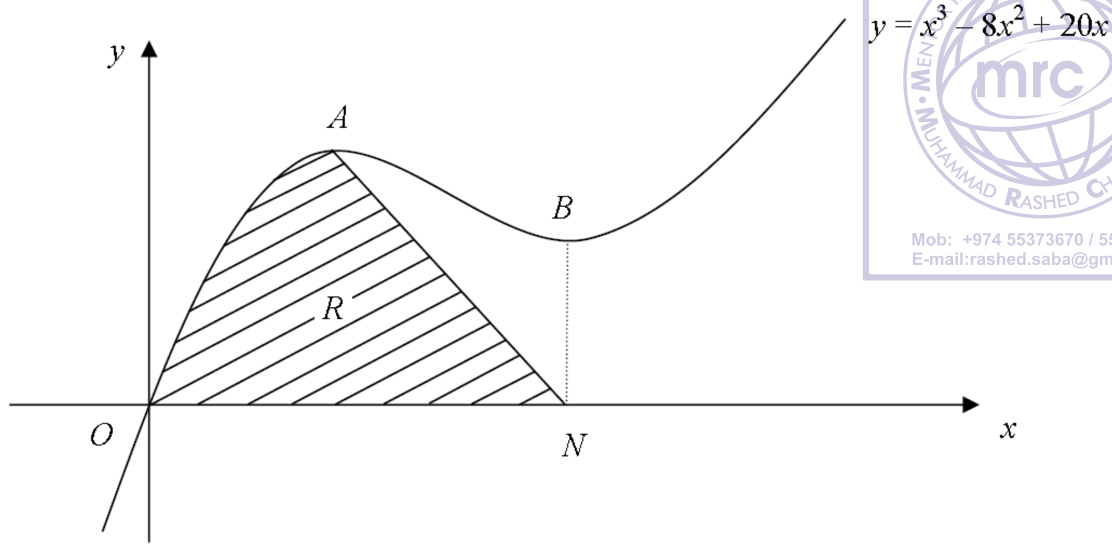


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$. The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N. The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\ dx$.

(3)

(d) Hence calculate the exact area of R.

(5)

JU-6-10

33.

(a) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\,dx$$

giving each term in its simplest form.

(4)

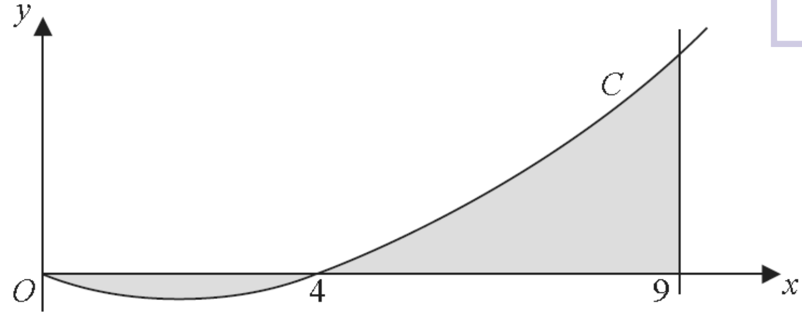


Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geqslant 0$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0).

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

JU-15-6

34.

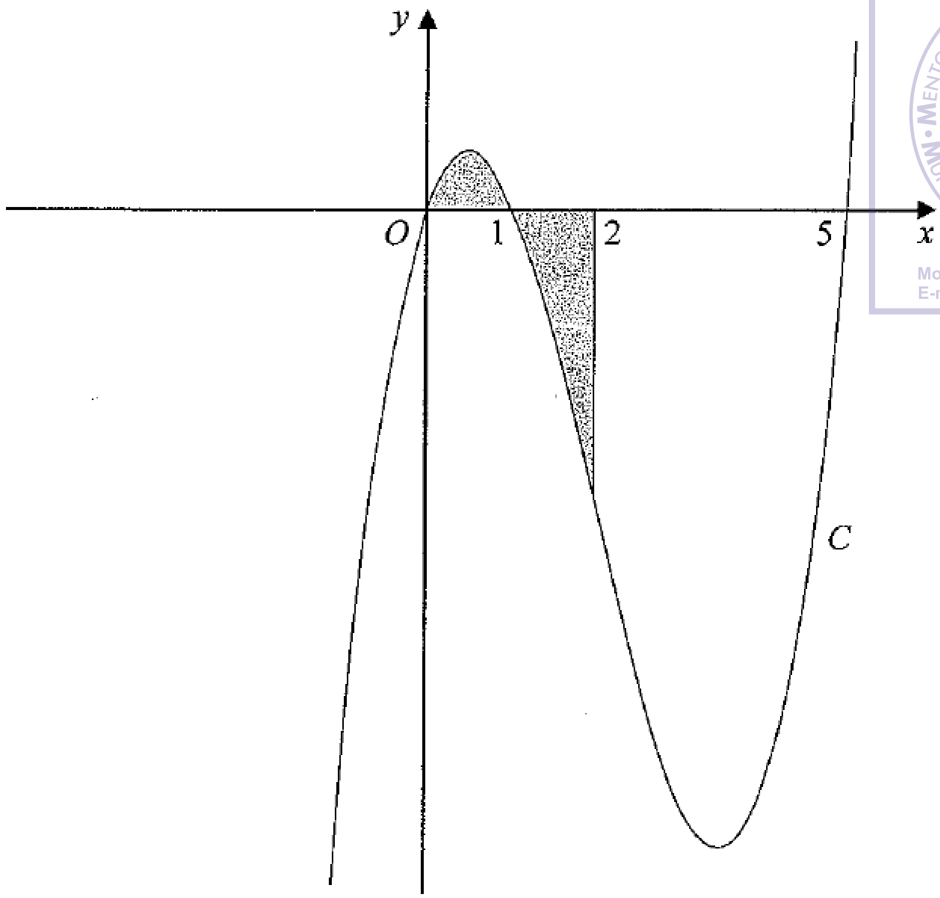


Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x-1)(x-5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

JA-7-7

35.

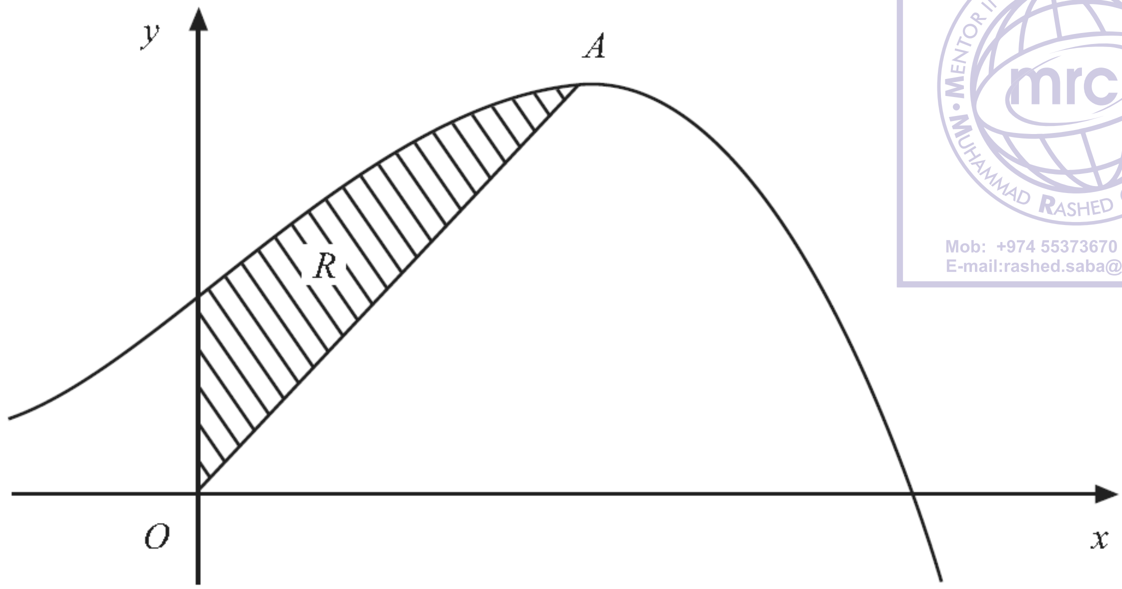


Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

JU-8-8

36.

(a) Complete the table below, giving values of $\sqrt{(2^x+1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x+1)}$	1.414	1.554	1.732	1.957			3

(2)

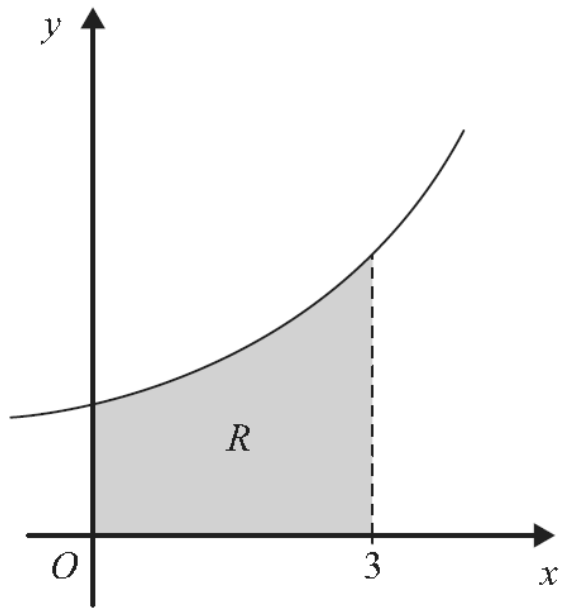


Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x+1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R.

(2)

JU-9-4

37.

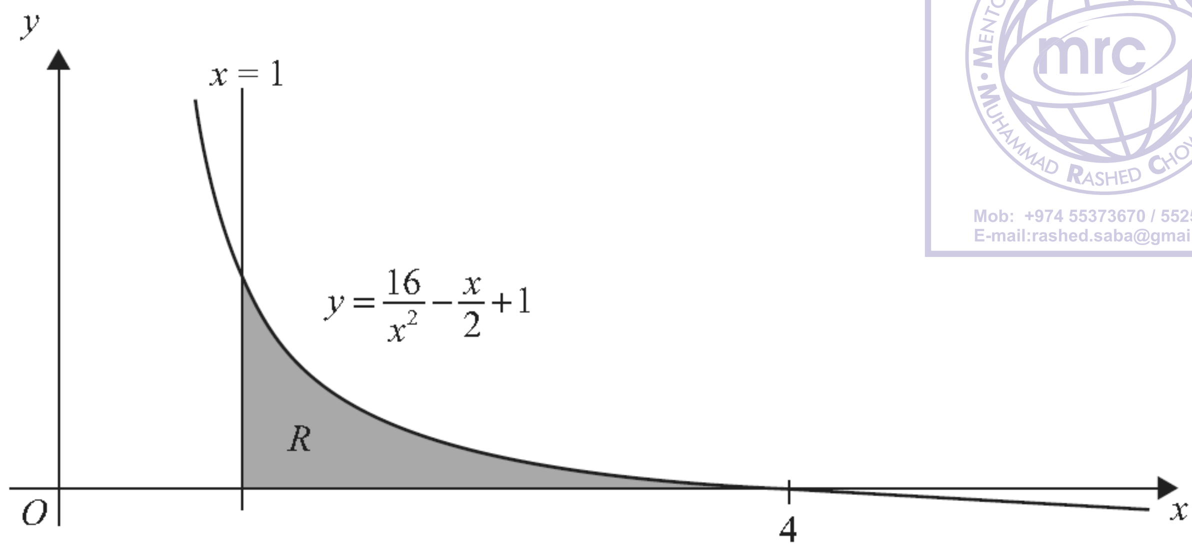


Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \qquad x > 0$$

The finite region R, bounded by the lines $x = 1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x = 2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

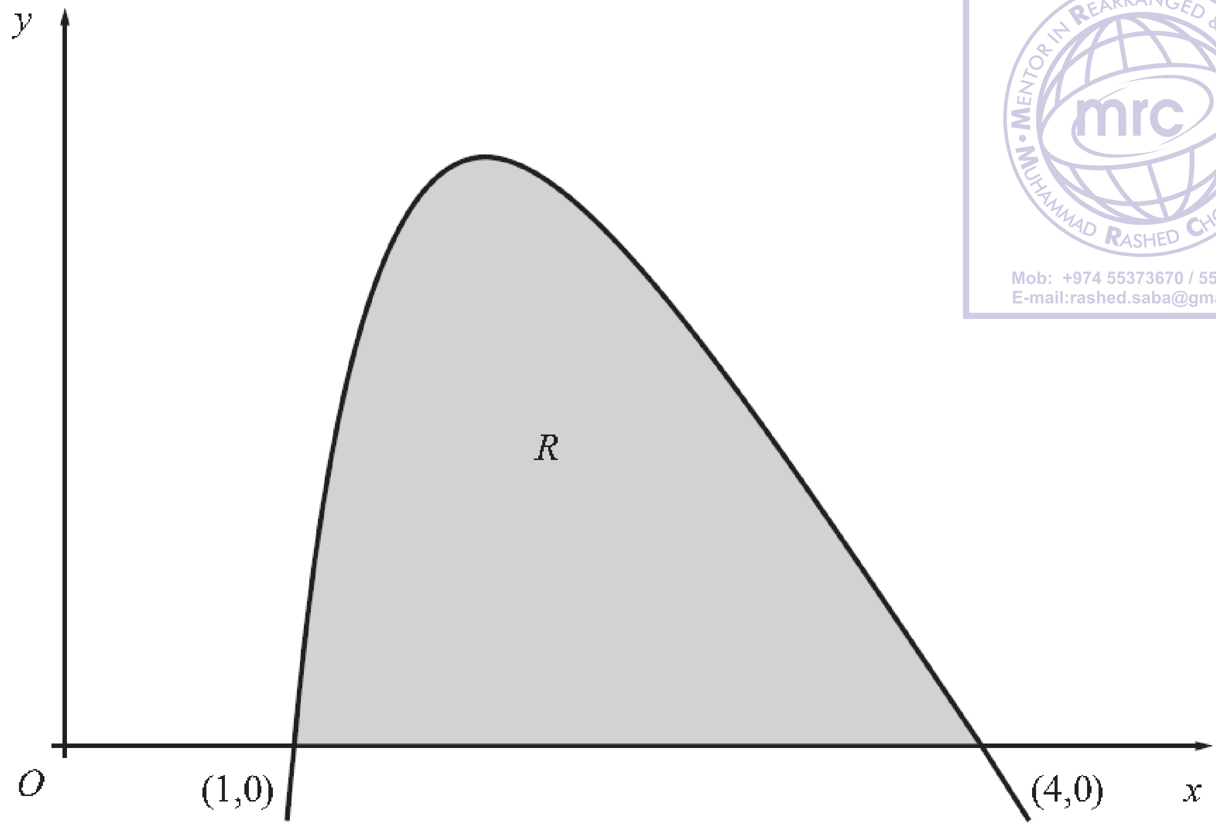
(4)

(c) Use integration to find the exact value for the area of R.

(5)

JA-12-6

38.



The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(a) Complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(6)

JA-13-9

39.

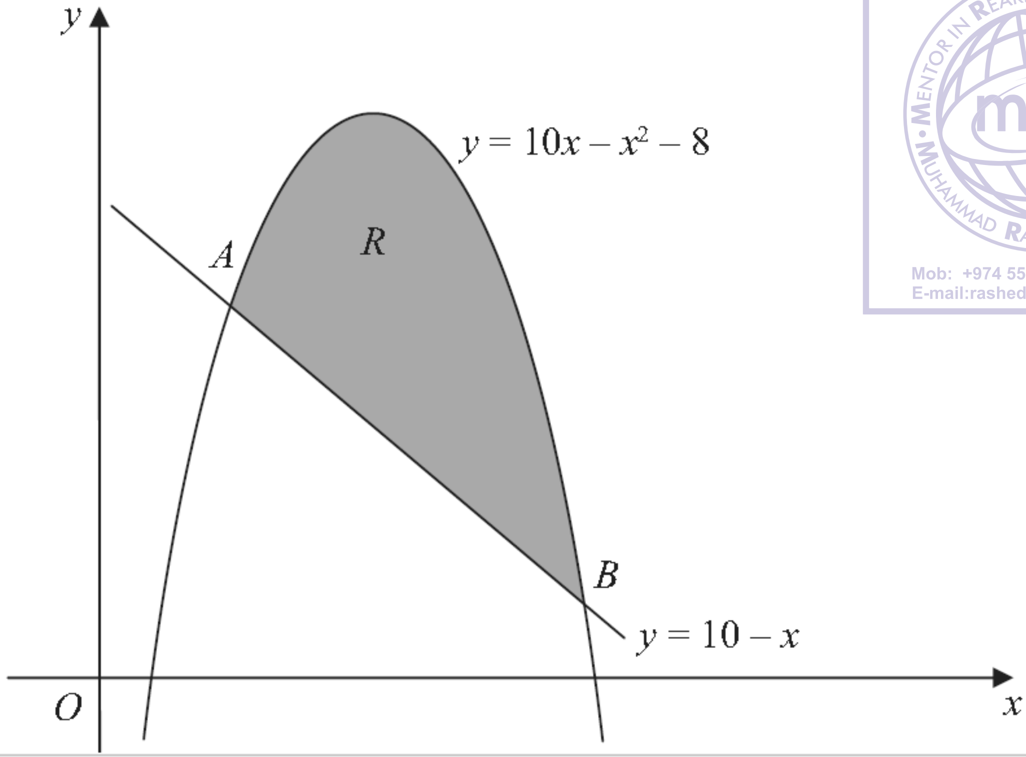


Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

ju-12-5

40.

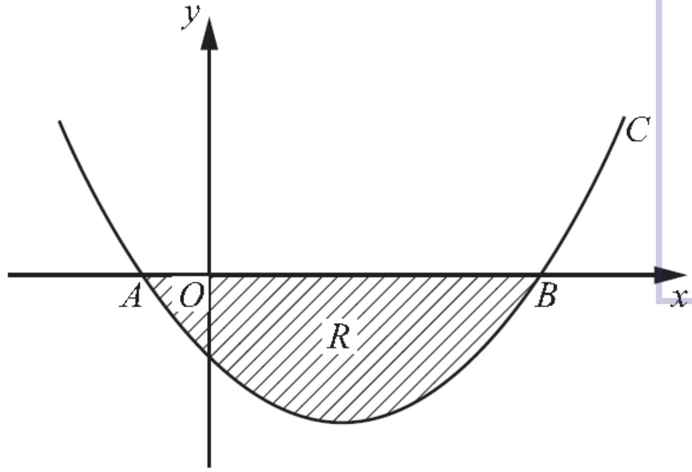


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

JA-11-4

41.

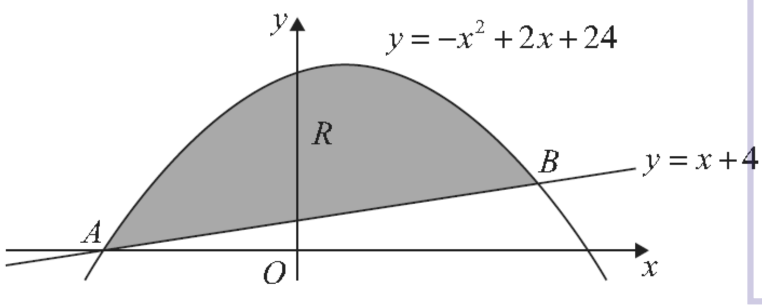


Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

JU-11-9