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International Examinations Papers

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Mechanics

TOPIC- Kinematics

Differentiation-Integration

Kinematics- Differentiation ↔ Integration

01 A particle P is held at rest at a fixed point O and then released. P falls freely under gravity until it reaches the point A which is 1.25 m below O.

(i) Find the speed of P at A and the time taken for P to reach A. [3]

The particle continues to fall, but now its downward acceleration t seconds after passing through A is $(10 - 0.3t)$ m s^{-2}.

(ii) Find the total distance P has fallen, 3 s after being released from O. [7]

02 A particle P starts from rest at the point A at time $t = 0$, where t is in seconds, and moves in a straight line with constant acceleration $a\,\text{m s}^{-2}$ for 10 s. For $10 \leqslant t \leqslant 20$, P continues to move along the line with velocity $v\,\text{m s}^{-1}$, where $v = \dfrac{800}{t^2} - 2$. Find

(i) the speed of P when $t = 10$, and the value of a, [2]

(ii) the value of t for which the acceleration of P is $-a\,\text{m s}^{-2}$, [4]

(iii) the displacement of P from A when $t = 20$. [6]

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03 A particle P travels in a straight line from A to D, passing through the points B and C. For the section AB the velocity of the particle is $(0.5t - 0.01t^2)$ m s^{-1}, where t s is the time after leaving A.

(i) Given that the acceleration of P at B is 0.1 m s^{-2}, find the time taken for P to travel from A to B. [3]

The acceleration of P from B to C is constant and equal to 0.1 m s^{-2}.

(ii) Given that P reaches C with speed 14 m s^{-1}, find the time taken for P to travel from B to C. [3]

P travels with constant deceleration 0.3 m s^{-2} from C to D. Given that the distance CD is 300 m, find

(iii) the speed with which P reaches D, [2]

(iv) the distance AD. [6]

4 A particle moves in a straight line. Its displacement t seconds after leaving the fixed point O is x metres, where $x = \frac{1}{2}t^2 + \frac{1}{30}t^3$. Find

(i) the speed of the particle when $t = 10$, [3]

(ii) the value of t for which the acceleration of the particle is twice its initial acceleration. [3]

5 A particle P moves along the x-axis in the positive direction. The velocity of P at time t s is $0.03t^2$ m s^{-1}. When $t = 5$ the displacement of P from the origin O is 2.5 m.

(i) Find an expression, in terms of t, for the displacement of P from O. [4]

(ii) Find the velocity of P when its displacement from O is 11.25 m. [3]

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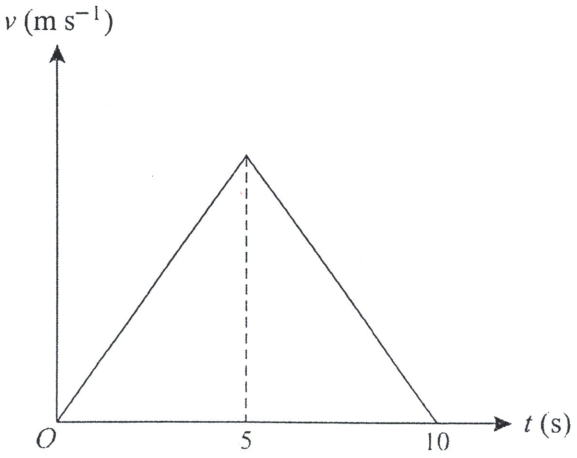

A particle P starts from rest at the point A and travels in a straight line, coming to rest again after 10 s. The velocity-time graph for P consists of two straight line segments (see diagram). A particle Q starts from rest at A at the same instant as P and travels along the same straight line as P. The velocity of Q is given by $v = 3t - 0.3t^2$ for $0 \leqslant t \leqslant 10$. The displacements from A of P and Q are the same when $t = 10$.

(i) Show that the greatest velocity of P during its motion is $10\ \text{m s}^{-1}$. [6]

(ii) Find the value of t, in the interval $0 < t < 5$, for which the acceleration of Q is the same as the acceleration of P. [3]

7 A particle P travels in a straight line. It passes through the point O of the line with velocity $5\,\text{m s}^{-1}$ at time $t = 0$, where t is in seconds. P's velocity after leaving O is given by

$$(0.002t^3 - 0.12t^2 + 1.8t + 5)\,\text{m s}^{-1}.$$

The velocity of P is increasing when $0 < t < T_1$ and when $t > T_2$, and the velocity of P is decreasing when $T_1 < t < T_2$.

(i) Find the values of T_1 and T_2 and the distance OP when $t = T_2$. [7]

(ii) Find the velocity of P when $t = T_2$ and sketch the velocity-time graph for the motion of P. [3]

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A particle P starts from rest at O and travels in a straight line. Its velocity v m s^{-1} at time t s is given by $v = 8t - 2t^2$ for $0 \leqslant t \leqslant 3$, and $v = \frac{54}{t^2}$ for $t > 3$. Find

(i) the distance travelled by P in the first 3 seconds, [4]

(ii) an expression in terms of t for the displacement of P from O, valid for $t > 3$, [3]

(iii) the value of v when the displacement of P from O is 27 m. [3]

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09 A particle P moves on a straight line. It starts at a point O on the line and returns to O 100 s later. The velocity of P is v m s^{-1} at time t s after leaving O, where

$$v = 0.0001t^3 - 0.015t^2 + 0.5t.$$

(i) Show that P is instantaneously at rest when $t = 0$, $t = 50$ and $t = 100$. [2]

(ii) Find the values of v at the times for which the acceleration of P is zero, and sketch the velocity-time graph for P's motion for $0 \leqslant t \leqslant 100$. [7]

(iii) Find the greatest distance of P from O for $0 \leqslant t \leqslant 100$. [4]

10 A particle travels in a straight line from A to B in 20 s. Its acceleration t seconds after leaving A is a m s^{-2}, where $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$. It is given that the particle comes to rest at B.

(i) Show that the initial speed of the particle is zero. [4]

(ii) Find the maximum speed of the particle. [2]

(iii) Find the distance AB. [4]

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11 A particle P moves in a straight line. At time t s, the displacement of P from O is s m and the acceleration of P is a m s^{-2}, where $a = 6t - 2$. When $t = 1$, $s = 7$ and when $t = 3$, $s = 29$.

(i) Find the set of values of t for which the particle is decelerating. [2]

(ii) Find s in terms of t. [5]

(iii) Find the time when the velocity of the particle is 10 m s^{-1}. [3]

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12 Alan starts walking from a point O, at a constant speed of $4\,\text{m s}^{-1}$, along a horizontal path. Ben walks along the same path, also starting from O. Ben starts from rest 5 s after Alan and accelerates at $1.2\,\text{m s}^{-2}$ for 5 s. Ben then continues to walk at a constant speed until he is at the same point, P, as Alan.

(i) Find how far Ben has travelled when he has been walking for 5 s and find his speed at this instant. [2]

(ii) Find the distance OP. [3]

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13 A particle P moves in a straight line, starting from the point O with velocity $2\,\text{m s}^{-1}$. The acceleration of P at time t s after leaving O is $2t^{\frac{2}{3}}\,\text{m s}^{-2}$.

(i) Show that $t^{\frac{5}{3}} = \frac{5}{6}$ when the velocity of P is $3\,\text{m s}^{-1}$. [4]

(ii) Find the distance of P from O when the velocity of P is $3\,\text{m s}^{-1}$. [3]

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14 A particle P moves in a straight line starting from a point O. At time t s after leaving O, the velocity, v m s^{-1}, of P is given by $v = (2t - 5)^3$.

(i) Find the values of t when the acceleration of P is 54 m s^{-2}. [3]

(ii) Find an expression for the displacement of P from O at time t s. [3]

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15 A particle P moves in a straight line. It starts from rest at A and comes to rest instantaneously at B. The velocity of P at time t seconds after leaving A is $v\,\text{m s}^{-1}$, where $v = 6t^2 - kt^3$ and k is a constant.

(i) Find an expression for the displacement of P from A in terms of t and k. [2]

(ii) Find an expression for t in terms of k when P is at B. [1]

Given that the distance AB is 108 m, find

(iii) the value of k, [2]

(iv) the maximum value of v when the particle is moving from A towards B. [3]

16 A particle P moves in a straight line passing through a point O. At time t s, the velocity of P, v m s^{-1}, is given by $v = qt + rt^2$, where q and r are constants. The particle has velocity 4 m s^{-1} when $t = 1$ and when $t = 2$.

(i) Show that, when $t = 0.5$, the acceleration of P is 4 m s^{-2}. [4]

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(ii) Find the values of t when P is at instantaneous rest. [2]

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(iii) The particle is at O when $t = 3$. Find the distance of P from O when $t = 0$. [4]

17 A racing car is moving in a straight line. The acceleration a m s^{-2} at time t s after the car starts from rest is given by

$$a = 15t - 3t^2 \qquad \text{for } 0 \leqslant t \leqslant 5,$$

$$a = -\frac{625}{t^2} \qquad \text{for } 5 < t \leqslant k,$$

where k is a constant.

(i) Find the maximum acceleration of the car in the first five seconds of its motion. [3]

(ii) Find the distance of the car from its starting point when $t = 5$. [3]

(iii) The car comes to rest when $t = k$. Find the value of k. [5]

18 A walker travels along a straight road passing through the points A and B on the road with speeds $0.9\,\text{m s}^{-1}$ and $1.3\,\text{m s}^{-1}$ respectively. The walker's acceleration between A and B is constant and equal to $0.004\,\text{m s}^{-2}$.

(i) Find the time taken by the walker to travel from A to B, and find the distance AB. [3]

A cyclist leaves A at the same instant as the walker. She starts from rest and travels along the straight road, passing through B at the same instant as the walker. At time t s after leaving A the cyclist's speed is $kt^3\,\text{m s}^{-1}$, where k is a constant.

(ii) Show that when $t = 64.05$ the speed of the walker and the speed of the cyclist are the same, correct to 3 significant figures. [5]

(ii) Find the cyclist's acceleration at the instant she passes through B. [2]

19 A vehicle starts from rest at a point O and moves in a straight line. Its speed v m s^{-1} at time t seconds after leaving O is defined as follows.

$$\text{For } 0 \leqslant t \leqslant 60, \quad v = \kappa_1 t - 0.005t^2,$$

$$\text{for } t \geqslant 60, \quad v = \frac{\kappa_2}{\sqrt{t}}.$$

The distance travelled by the vehicle during the first 60 s is 540 m.

(i) Find the value of the constant κ_1 and show that $\kappa_2 = 12\sqrt{(60)}$. [5]

(ii) Find an expression in terms of t for the total distance travelled when $t \geqslant 60$. [2]

(iii) Find the speed of the vehicle when it has travelled a total distance of 1260 m. [3]

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2 A particle P moves in a straight line, starting from a point O. At time t s after leaving O, the velocity of P, v m s^{-1}, is given by $v = 4t^2 - 8t + 3$.

(i) Find the two values of t at which P is at instantaneous rest. [2]

(ii) Find the distance travelled by P between these two times. [3]

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21 A particle moves in a straight line. Its velocity t seconds after leaving a fixed point O on the line is v m s^{-1}, where $v = 0.2t + 0.006t^2$. For the instant when the acceleration of the particle is 2.5 times its initial acceleration,

(i) show that $t = 25$, [3]

(ii) find the displacement of the particle from O. [3]

2 A particle moves in a straight line. Its displacement t s after leaving a fixed point O on the line is s m, where $s = 2t^2 - \frac{80}{3}t^{\frac{3}{2}}$.

(i) Find the time at which the acceleration of the particle is zero. [4]

(ii) Find the displacement and velocity of the particle at this instant. [2]

23 A particle A moves in a straight line with constant speed $10\,\text{m s}^{-1}$. Two seconds after A passes a point O on the line, a particle B passes through O, moving along the line in the same direction as A. Particle B has speed $16\,\text{m s}^{-1}$ at O and has a constant deceleration of $2\,\text{m s}^{-2}$.

(i) Find expressions, in terms of t, for the displacement from O of each particle t s after B passes through O. [3]

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(ii) Find the distance between the particles when B comes to instantaneous rest. [3]

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(iii) Find the minimum distance between the particles. [3]

24 An aeroplane moves along a straight horizontal runway before taking off. It starts from rest at O and has speed $90\,\text{m s}^{-1}$ at the instant it takes off. While the aeroplane is on the runway at time t seconds after leaving O, its acceleration is $(1.5 + 0.012t)\,\text{m s}^{-2}$. Find

(i) the value of t at the instant the aeroplane takes off, [4]

(ii) the distance travelled by the aeroplane on the runway. [3]

25 A particle P moves in a straight line. P starts from rest at O and travels to A where it comes to rest, taking 50 seconds. The speed of P at time t seconds after leaving O is v m s^{-1}, where v is defined as follows.

$$\text{For } 0 \leqslant t \leqslant 5, \quad v = t - 0.1t^2,$$
$$\text{for } 5 \leqslant t \leqslant 45, \quad v \text{ is constant},$$
$$\text{for } 45 \leqslant t \leqslant 50, \quad v = 9t - 0.1t^2 - 200.$$

(i) Find the distance travelled by P in the first 5 seconds. [3]

(ii) Find the total distance from O to A, and deduce the average speed of P for the whole journey from O to A. [6]

26 A particle P moves in a straight line, starting from a point O. The velocity of P, measured in m s^{-1}, at time t s after leaving O is given by

$$v = 0.6t - 0.03t^2.$$

(i) Verify that, when $t = 5$, the particle is 6.25 m from O. Find the acceleration of the particle at this time. [4]

(ii) Find the values of t at which the particle is travelling at half of its maximum velocity. [6]

27 A particle P starts from rest at a point O and moves in a straight line. P has acceleration 0.6τ m s^{-2} at time τ seconds after leaving O, until $\tau = 10$.

(i) Find the velocity and displacement from O of P when $\tau = 10$. [5]

After $\tau = 10$, P has acceleration -0.4τ m s^{-2} until it comes to rest at a point A.

(ii) Find the distance OA. [7]

28 A particle P starts to move from a point O and travels in a straight line. The velocity of P is $k(60t^2 - t^3)$ m s^{-1} at time t s after leaving O, where k is a constant. The maximum velocity of P is 6.4 m s^{-1}.

(i) Show that $k = 0.0002$. [3]

P comes to instantaneous rest at a point A on the line. Find

(ii) the distance OA, [5]

(iii) the magnitude of the acceleration of P at A, [2]

(iv) the speed of P when it subsequently passes through O. [2]

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29 Two cyclists P and Q travel along a straight road ABC, starting simultaneously at A and arriving simultaneously at C. Both cyclists pass through B 400 s after leaving A. Cyclist P starts with speed $3\ \text{m s}^{-1}$ and increases this speed with constant acceleration $0.005\ \text{m s}^{-2}$ until he reaches B.

(i) Show that the distance AB is 1600 m and find P's speed at B. [3]

Cyclist Q travels from A to B with speed $v\ \text{m s}^{-1}$ at time t seconds after leaving A, where

$$v = 0.04t - 0.0001t^2 + k,$$

and k is a constant.

(ii) Find the value of k and the maximum speed of Q before he has reached B. [6]

Cyclist P travels from B to C, a distance of 1400 m, at the speed he had reached at B. Cyclist Q travels from B to C with constant acceleration $a\ \text{m s}^{-2}$.

(iii) Find the time taken for the cyclists to travel from B to C and find the value of a. [4]

30 A particle P travels from a point O along a straight line and comes to instantaneous rest at a point A. The velocity of P at time t s after leaving O is v m s^{-1}, where $v = 0.027(10t^2 - t^3)$. Find

(i) the distance OA, [4]

(ii) the maximum velocity of P while moving from O to A. [3]

31 A particle P moves in a straight line. It starts from a point O on the line with velocity $1.8\,\text{m s}^{-1}$. The acceleration of P at time t s after leaving O is $0.8t^{-0.75}\,\text{m s}^{-2}$. Find the displacement of P from O when $t = 16$. [6]

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32 A particle P moves along a straight line for 100 s. It starts at a point O and at time t seconds after leaving O the velocity of P is v m s^{-1}, where

$$v = 0.000\,04t^3 - 0.006t^2 + 0.288t.$$

(i) Find the values of t at which the acceleration of P is zero. [3]

(ii) Find the displacement of P from O when $t = 100$. [3]

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3 3 A particle P starts at the point O and travels in a straight line. At time t seconds after leaving O the velocity of P is v m s^{-1}, where $v = 0.75t^2 - 0.0625t^3$. Find

(i) the positive value of t for which the acceleration is zero, [3]

(ii) the distance travelled by P before it changes its direction of motion. [5]

34 A particle P starts from rest and moves in a straight line for 18 seconds. For the first 8 s motion P has constant acceleration $0.25\,\text{m s}^{-2}$. Subsequently P's velocity, $v\,\text{m s}^{-1}$ at time after the motion started, is given by

$$v = -0.1t^2 + 2.4t - k,$$

where $8 \leqslant t \leqslant 18$ and k is a constant.

(i) Find the value of v when $t = 8$ and hence find the value of k. [2]

(ii) Find the maximum velocity of P. [2]

(iii) Find the displacement of P from its initial position when $t = 18$. [3]

35 Two particles P and Q are projected vertically upwards from horizontal ground at the same instant. The speeds of projection of P and Q are $12\,\text{m s}^{-1}$ and $7\,\text{m s}^{-1}$ respectively and the heights of P and Q above the ground, t seconds after projection, are h_P m and h_Q m respectively. Each particle comes to rest on returning to the ground.

(i) Find the set of values of t for which the particles are travelling in opposite directions. [3]

(ii) At a certain instant, P and Q are above the ground and $3h_P = 8h_Q$. Find the velocities of P and Q at this instant. [5]

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36 A particle travels in a straight line from a point P to a point Q. Its velocity t seconds after leaving P is $v\,\text{m s}^{-1}$, where $v = 4t - \frac{1}{16}t^3$. The distance PQ is 64 m.

(i) Find the time taken for the particle to travel from P to Q. [5]

(ii) Find the set of values of t for which the acceleration of the particle is positive. [4]

37 A car driver makes a journey in a straight line from A to B, starting from rest. The speed of the car increases to a maximum, then decreases until the car is at rest at B. The distance travelled by the car t seconds after leaving A is $0.000\,011\,7(400t^3 - 3t^4)$ metres.

(i) Find the distance AB. [3]

(ii) Find the maximum speed of the car. [4]

(iii) Find the acceleration of the car

(a) as it starts from A,

(b) as it arrives at B.

[2]

(iv) Sketch the velocity-time graph for the journey. [2]

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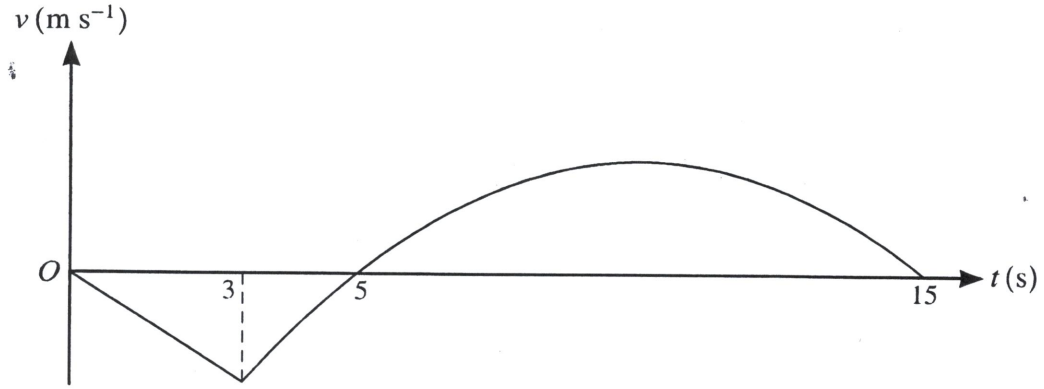

The diagram shows the velocity-time graph for the motion of a particle P which moves on a straight line BAC. It starts at A and travels to B taking 5 s. It then reverses direction and travels from B to C taking 10 s. For the first 3 s of P's motion its acceleration is constant. For the remaining 12 s the velocity of P is v m s^{-1} at time t s after leaving A, where

$$v = -0.2t^2 + 4t - 15 \quad \text{for } 3 \leqslant t \leqslant 15.$$

(i) Find the value of v when $t = 3$ and the magnitude of the acceleration of P for the first 3 s of its motion. [3]

(ii) Find the maximum velocity of P while it is moving from B to C. [3]

(iii) Find the average speed of P,

(a) while moving from A to B,

(b) for the whole journey. [6]

39 Two particles A and B start to move at the same instant from a point O. The particles move in the same direction along the same straight line. The acceleration of A at time t s after starting to move is $a\ m\ s^{-2}$, where $a = 0.05 - 0.0002t$.

(i) Find A's velocity when $t = 200$ and when $t = 500$. [4]

B moves with constant acceleration for the first 200 s and has the same velocity as A when $t = 200$. B moves with constant retardation from $t = 200$ to $t = 500$ and has the same velocity as A when $t = 500$.

(ii) Find the distance between A and B when $t = 500$. [6]

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40 A particle P starts from rest at a point O of a straight line and moves along the line. The displacement of the particle at time t s after leaving O is x m, where

$$x = 0.08t^2 - 0.0002t^3.$$

(i) Find the value of t when P returns to O and find the speed of P as it passes through O on its return. [4]

(ii) For the motion of P until the instant it returns to O, find

(a) the total distance travelled, [3]

(b) the average speed. [2]

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41 A particle P moves on a straight line, starting from rest at a point O of the line. The time after P starts to move is t s, and the particle moves along the line with constant acceleration $\frac{1}{4}$ m s^{-2} until it passes through a point A at time $t = 8$. After passing through A the velocity of P is $\frac{1}{2}t^{\frac{2}{3}}$ m s^{-1}.

(i) Find the acceleration of P immediately after it passes through A. Hence show that the acceleration of P decreases by $\frac{1}{12}$ m s^{-2} as it passes through A. [4]

(ii) Find the distance moved by P from $t = 0$ to $t = 27$. [3]

42 A particle P starts from a fixed point O and moves in a straight line. At time t s after leaving O, the velocity v m s^{-1} of P is given by $v = 6t - 0.3t^2$. The particle comes to instantaneous rest at point X.

(i) Find the distance OX. [4]

A second particle Q starts from rest from O, at the same instant as P, and also travels in a straight line. The acceleration a m s^{-2} of Q is given by $a = k - 12t$, where k is a constant. The displacement of Q from O is 400 m when $t = 10$.

(ii) Find the value of k. [4]

43 A particle P moves in a straight line. It starts at a point O on the line and at time t s after leaving O it has a velocity v m s^{-1}, where $v = 6t^2 - 30t + 24$.

(i) Find the set of values of t for which the acceleration of the particle is negative. [2]

(ii) Find the distance between the two positions at which P is at instantaneous rest. [4]

(iii) Find the two positive values of t at which P passes through O. [3]

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