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International Examinations Papers

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Pure Mathematics-1

TOPIC- FUNCTIONS

FUNCTIONS

1 The function f is defined by $f : x \mapsto 2x + k$, $x \in \mathbb{R}$, where k is a constant.

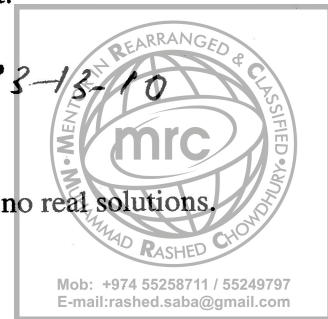   (I) In the case where $k = 3$, solve the equation $ff(x) = 25$.   [2]

The function g is defined by $g : x \mapsto x^2 - 6x + 8$, $x \in \mathbb{R}$.

   (II) Find the set of values of k for which the equation $f(x) = g(x)$ has no real solutions.   [3]

The function h is defined by $h : x \mapsto x^2 - 6x + 8$, $x > 3$.

   (III) Find an expression for $h^{-1}(x)$.   [4]

2 A curve has equation $y = f(x)$. It is given that $f'(x) = x^{-\frac{3}{2}} + 1$ and that $f(4) = 5$. Find $f(x)$. [4]

N-13-13-2

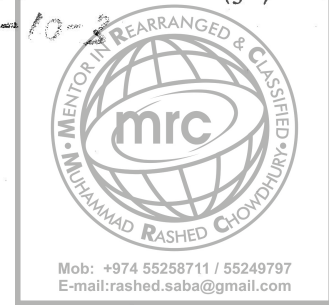**3** The function $f : x \mapsto a + b \cos x$ is defined for $0 \leqslant x \leqslant 2\pi$. Given that $f(0) = 10$ and that $f\left(\frac{2}{3}\pi\right) = 1$, find

   (i) the values of a and b, [2]

  (ii) the range of f, [1]
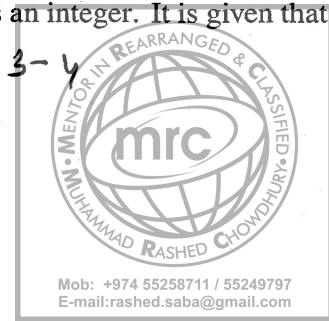
 (iii) the exact value of $f\left(\frac{5}{6}\pi\right)$. [2]

4 The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n. $N-16-3-4$ [4]

5 The function f is defined by $f : x \mapsto ax + b$, for $x \in \mathbb{R}$, where a and b are constants. It is given that $f(2) = 1$ and $f(5) = 7$.

   (i) Find the values of a and b. [2]

   (ii) Solve the equation $ff(x) = 0$. [3]

6 The function f is defined by $f : x \mapsto \dfrac{x+3}{2x-1}, \ x \in \mathbb{R}, \ x \neq \frac{1}{2}.$

$\mathcal{J}$- 11-12- 6

   (i) Show that $ff(x) = x$. [3]

   (ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$. [2]
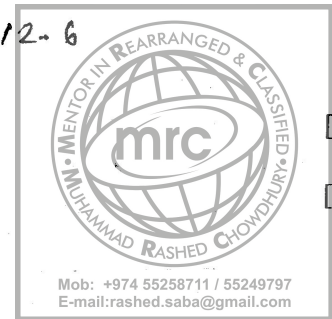
07   The function f is defined by

$$f : x \mapsto x^2 + 1 \ \text{ for } \ x \geqslant 0.$$

    (i) Define in a similar way the inverse function f^{-1}.   [3]

    (ii) Solve the equation $ff(x) = \frac{185}{16}$.   [3]
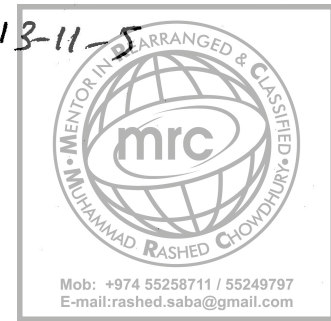
8 The function f is defined by $f(x) = 3x + 1$ for $x \leqslant a$, where a is a constant. The function g is defined by $g(x) = -1 - x^2$ for $x \leqslant -1$.

  (i) Find the largest value of a for which the composite function gf can be formed. [2]

For the case where $a = -1$,

  (ii) solve the equation $fg(x) + 14 = 0$, [3]

  (iii) find the set of values of x which satisfy the inequality $gf(x) \leqslant -50$. [4]

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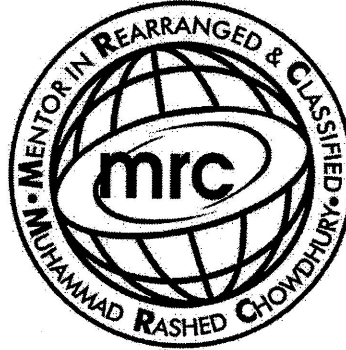
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Pure

Mathematics-1

TOPIC- FUNCTIONS

SUB-TOPIC- Quadratics

FUNCTIONS

1 The function f is defined by $f : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

   (i) Find the set of values of x for which $f(x) \leqslant 3$. [3]

   (ii) Given that the line $y = mx + c$ is a tangent to the curve $y = f(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $g : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

   (iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

   (iv) State the smallest value of k for which g has an inverse. [1]

   (v) For this value of k, find an expression for $g^{-1}(x)$. [2]

0 2 The function f : $x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$.

[4]

(ii) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants.   [3]

(iii) Find the range of f.   [1]

The function g : $x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse.   [1]

(v) For this value of A, find an expression for $g^{-1}(x)$ in terms of x.   [3]
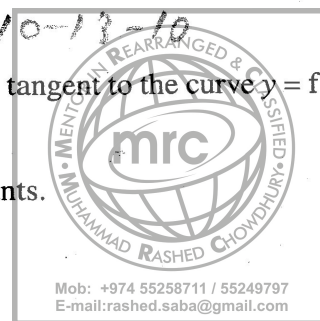
U 3 The function f is defined by $f(x) = 4x^2 - 24x + 11$, for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x - b)^2 + c$ and hence state the coordinates of the vertex of the graph of $y = f(x)$. [4]

The function g is defined by $g(x) = 4x^2 - 24x + 11$, for $x \leqslant 1$.

(ii) State the range of g. [2]

(iii) Find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. [4]

0 4   The function f is such that $f(x) = 8 - (x-2)^2$, for $x \in \mathbb{R}$.

   (i) Find the coordinates and the nature of the stationary point on the curve $y = f(x)$.   [3]

The function g is such that $g(x) = 8 - (x-2)^2$, for $k \leqslant x \leqslant 4$, where k is a constant.

   (ii) State the smallest value of k for which g has an inverse.   [1]

For this value of k,

   (iii) find an expression for $g^{-1}(x)$,   [3]

   (iv) sketch, on the same diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$.   [3]

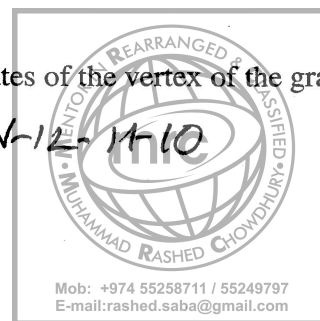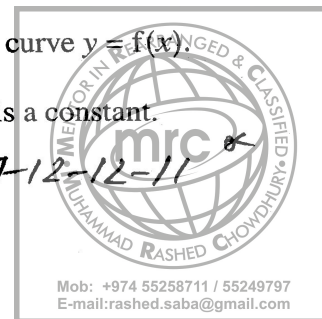**05** The function f is defined by $f : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

The function g is defined by $g : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$ and state the range of g^{-1}. [4]
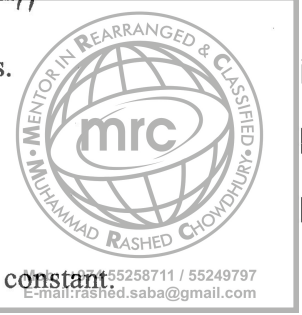
0 6 The function f is defined by $f : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

   (i) Find the set of values of x for which $f(x) > 4$. [3]

   (ii) Express $f(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. [2]

   (iii) Write down the range of f. [1]

   (iv) State, with a reason, whether f has an inverse. [1]

The function g is defined by $g : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

   (v) Solve the equation $g(x) = 10$. [3]

0 7   **(i)** Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a, b and c are constants.   [3]

   (ii) Functions f and g are both defined for $x > 0$. It is given that $f(x) = x^2 + 1$ and $fg(x) = 4x^2 + 12x + 10$. Find $g(x)$.   [1]

   (iii) Find $(fg)^{-1}(x)$ and give the domain of $(fg)^{-1}$.   [4]

N-17-13-8
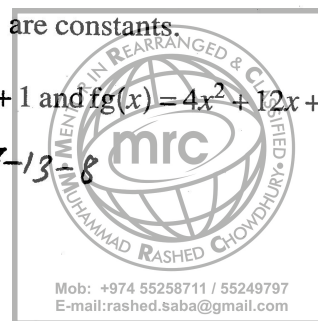
8 The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

    (i) Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants.   [2]

    (ii) State the range of f in terms of k.   [1]

    (iii) State the smallest value of p for which f is one-one.   [1]

    (iv) For the value of p found in part **(iii)**, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}, giving your answers in terms of k.   [4]
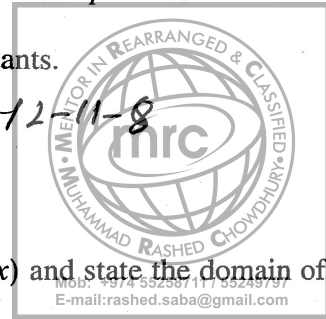
9 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

   (i) Express $f(x)$ in the form $a(x - b)^2 - c$. [3]

   (ii) State the range of f. [1]

   (iii) Find the set of values of x for which $f(x) < 21$. [3]

   The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

   (iv) Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]
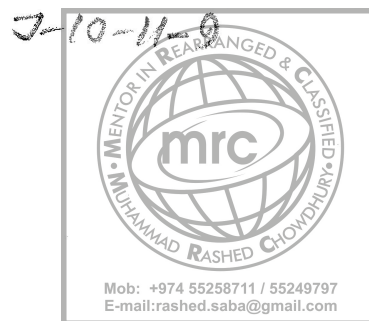
10 **(i)** Express $x^2 - 2x - 15$ in the form $(x + a)^2 + b$. [2]

The function f is defined for $p \leqslant x \leqslant q$, where p and q are positive constants, by

$$f : x \mapsto x^2 - 2x - 15.$$

The range of f is given by $c \leqslant f(x) \leqslant d$, where c and d are constants.

(ii) State the smallest possible value of c. [1]

For the case where $c = 9$ and $d = 65$,

(iii) find p and q, [4]

(iv) find an expression for $f^{-1}(x)$. [3]

N-14-11-10
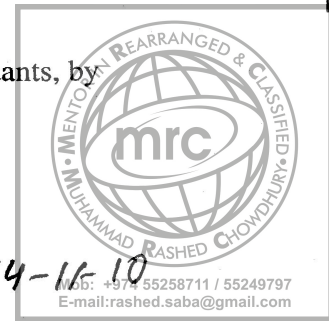
11 The function f is defined by $f : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of **p** for which the equation $f(x) = p$ has no real roots. [3]

The function g is defined by $g : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $g(x)$ in the form $a(x + b)^2 + c$, where **a**, **b** and **c** are constants. [3]

(iii) Find the range of g. [2]

The function h is defined by $h : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$, where **k** is a constant.

(iv) State the smallest value of **k** for which h has an inverse. [1]

(v) For this value of **k**, find an expression for $h^{-1}(x)$. [3]

12. The function f is defined by $f : x \mapsto 2x^2 - 12x + 13$ for $0 \leqslant x \leqslant A$, where A is a constant.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the value of A for which the graph of $y = f(x)$ has a line of symmetry. [1]

(iii) When A has this value, find the range of f. [2]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 13$ for $x \geqslant 4$.

(iv) Explain why g has an inverse. [1]

(v) Obtain an expression, in terms of x, for $g^{-1}(x)$. [3]
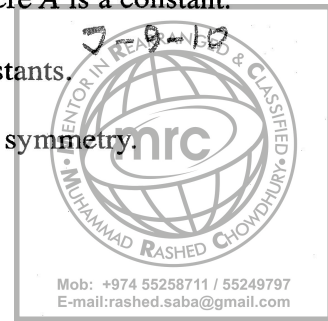
13 **(i)** Express $-x^2 + 6x - 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

The function f : $x \mapsto -x^2 + 6x - 5$ is defined for $x \geqslant m$, where m is a constant.

(ii) State the smallest value of m for which f is one-one. [1]

(iii) For the case where $m = 5$, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]
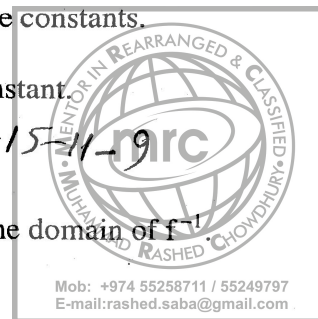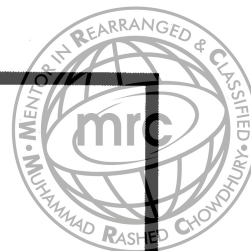
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Mathematics-1

TOPIC- FUNCTIONS

SUB-TOPIC- Domain-Range

FUNCTIONS

1) The function f is such that $f(x) = 2x + 3$ for $x \geqslant 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leqslant q$, where a, b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leqslant q$.

   (i) Find the values of a and b.   [3]

   (ii) Find the greatest possible value of q.   [2]

It is now given that $q = -3$.

   (iii) Find the range of fg.   [1]

   (iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$.   [3]
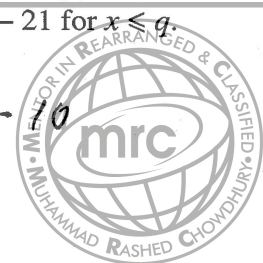
2 A function f is such that $f(x) = \sqrt{\left(\dfrac{x+3}{2}\right)} + 1$, for $x \geqslant -3$. Find   N-12-12-2

    (i) $f^{-1}(x)$ in the form $ax^2 + bx + c$, where a, b and c are constants,   [3]

    (ii) the domain of f^{-1}.   [1]
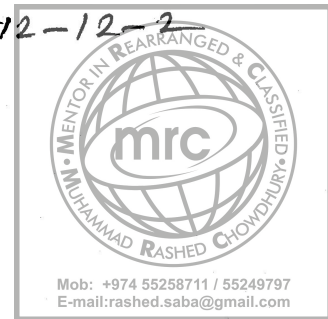
0 3 The function f : $x \mapsto 2x - a$, where a is a constant, is defined for all real x.

(i) In the case where $a = 3$, solve the equation ff$(x) = 11$. [3]

The function g : $x \mapsto x^2 - 6x$ is defined for all real x.

(ii) Find the value of a for which the equation f$(x) = $ g(x) has exactly one real solution. [3]

The function h : $x \mapsto x^2 - 6x$ is defined for the domain $x \geqslant 3$.

(iii) Express $x^2 - 6x$ in the form $(x - p)^2 - q$, where p and q are constants. [2]

(iv) Find an expression for h$^{-1}(x)$ and state the domain of h^{-1}. [4]
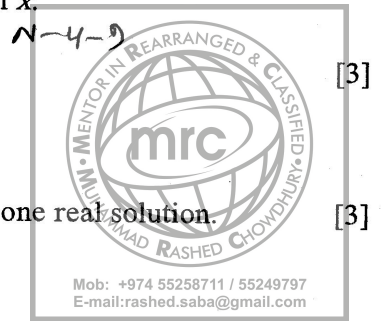
FUNCTIONS

04

(i) Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f is defined by $f(x) = 2x^2 - 12x + 13$ for $x \geqslant k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k. [1]

The value of k is now given to be 7.

(iii) Find the range of f. [1]

(iv) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [5]

5 A function f is such that $f(x) = \dfrac{15}{2x+3}$ for $0 \leqslant x \leqslant 6$.

J-14-13-5

(i) Find an expression for $f'(x)$ and use your result to explain why f has an inverse. [3]

(ii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1}. [4]
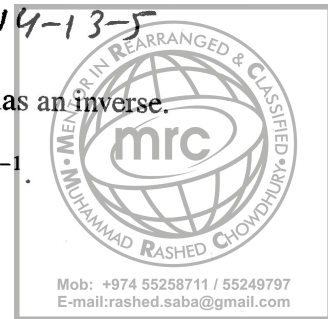
6 The function f is such that $f(x) = (3x+2)^3 - 5$ for $x \geqslant 0$.   J-8-6

(i) Obtain an expression for $f'(x)$ and hence explain why f is an increasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

7 The function f is defined by

$$f(x) = x^2 - 4x + 7 \quad \text{for } x > 2.$$

(i) Express $f(x)$ in the form $(x-a)^2 + b$ and hence state the range of f. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by

$$g(x) = x - 2 \quad \text{for } x > 2.$$

The function h is such that $f = hg$ and the domain of h is $x > 0$.

(iii) Obtain an expression for $h(x)$. [1]
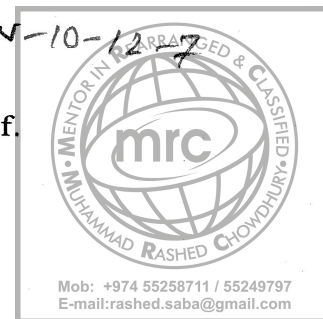
8 A function f is defined by $f : x \mapsto (2x - 3)^3 - 8$, for $2 \leqslant x \leqslant 4$.

N-5-8

(i) Find an expression, in terms of x, for $f'(x)$ and show that f is an increasing function. [4]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}. [4]
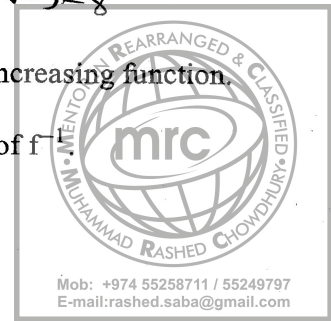
9 A function f is defined by $f(x) = \dfrac{5}{1 - 3x}$, for $x \geqslant 1$. $\mathcal{Z}/3{-}12{-}9$

   (i) Find an expression for $f'(x)$.   [2]

   (ii) Determine, with a reason, whether f is an increasing function, a decreasing function or neither.   [1]

   (iii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1}.   [5]
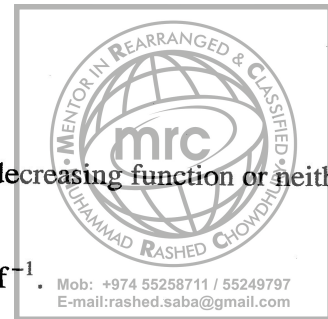
10 The function f is defined by $f : x \mapsto x^2 + 4x$ for $x \geqslant c$, where **c** is a constant. It is given that f is a one-one function.

N-13-13-10

(i) State the range of f in terms of **c** and find the smallest possible value of **c**. [3]

The function g is defined by $g : x \mapsto ax + b$ for $x \geqslant 0$, where **a** and **b** are positive constants. It is given that, when $c = 0$, $gf(1) = 11$ and $fg(1) = 21$.

(ii) Write down two equations in **a** and **b** and solve them to find the values of **a** and **b**. [6]
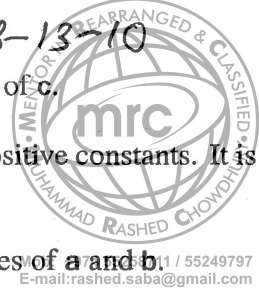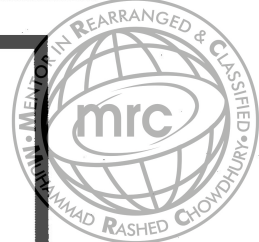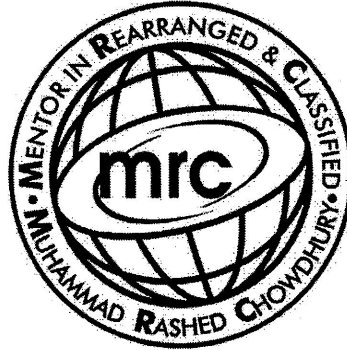
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Pure Mathematics-1

TOPIC- FUNCTIONS

SUB-TOPIC- Trigonometry

FUNCTIONS

1 The function f is defined by $f : x \mapsto 4\sin x - 1$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

   (i) State the range of f. [2]

   (ii) Find the coordinates of the points at which the curve $y = f(x)$ intersects the coordinate axes. [3]

   (iii) Sketch the graph of $y = f(x)$. [2]

   (iv) Obtain an expression for $f^{-1}(x)$, stating both the domain and range of f^{-1}. [4]
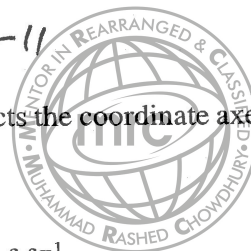
0 2 The function $f : x \mapsto 4 - 3\sin x$ is defined for the domain $0 \leqslant x \leqslant 2\pi$.

$J-10-12-11$

    (i) Solve the equation $f(x) = 2$.　　　　　　　　　　　　　　　　　[3]

    (ii) Sketch the graph of $y = f(x)$.　　　　　　　　　　　　　　　[2]

    (iii) Find the set of values of k for which the equation $f(x) = k$ has no solution.　[2]

The function $g : x \mapsto 4 - 3\sin x$ is defined for the domain $\frac{1}{2}\pi \leqslant x \leqslant A$.

    (iv) State the largest value of A for which g has an inverse.　　　　　[1]

    (v) For this value of A, find the value of $g^{-1}(3)$.　　　　　　　　[2]
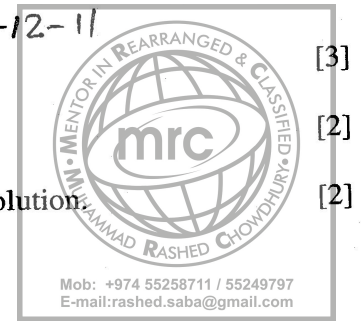
0 3   The function f is such that $f(x) = a - b\cos x$ for $0° \leqslant x \leqslant 360°$, where a and b are positive constants. The maximum value of $f(x)$ is 10 and the minimum value is –2. $N-8-5$

   (i) Find the values of a and b.   [3]

   (ii) Solve the equation $f(x) = 0$.   [3]

   (iii) Sketch the graph of $y = f(x)$.   [2]

4 The function f is defined by $f : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

   (i) Find the range of f. [2]

   (ii) Sketch the graph of $y = f(x)$. [3]

   (iii) State, with a reason, whether f has an inverse. [1]
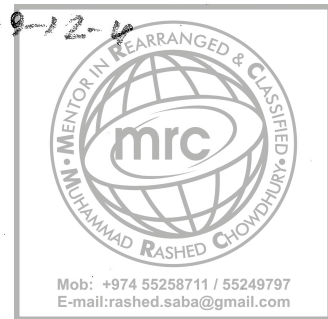
5 The function f is such that $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \leqslant x \leqslant \pi$.

   (i) Express $f(x)$ in the form $a + b\cos^2 x$, stating the values of a and b.   [2]

   (ii) State the greatest and least values of $f(x)$.   [2]

   (iii) Solve the equation $f(x) + 1 = 0$.   [3]

6 The function f : $x \mapsto 5\sin^2 x + 3\cos^2 x$ is defined for the domain $0 \leqslant x \leqslant \pi$.

   (i) Express $f(x)$ in the form $a + b\sin^2 x$, stating the values of a and b.   [2]

   (ii) Hence find the values of x for which $f(x) = 7\sin x$.   [3]

   (iii) State the range of f.   [2]

7 A function f is defined by $f : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant 360°$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

A function g is defined by $g : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant A°$, where A is a constant.

(iii) State the largest value of A for which g has an inverse. [1]

(iv) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$. [2]

8 The function $f : x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ is defined for $0 \leqslant x \leqslant 2\pi$. $J-15-11-8$

  (i) Solve the equation $f(x) = 7$, giving your answer correct to 2 decimal places.   [3]

  (ii) Sketch the graph of $y = f(x)$.   [2]

  (iii) Explain why f has an inverse.   [1]

  (iv) Obtain an expression for $f^{-1}(x)$.   [3]
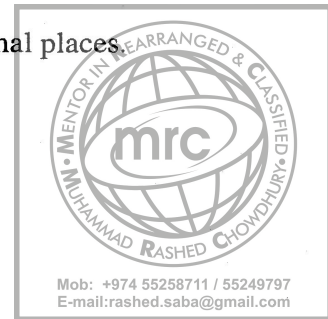
9 The function f is such that $f(x) = 3 - 4\cos^k x$, for $0 \leqslant x \leqslant \pi$, where k is a constant.

  (i) In the case where $k = 2$,

    (a) find the range of f,   [2]

    (b) find the exact solutions of the equation $f(x) = 1$.   [3]

  (ii) In the case where $k = 1$,

    (a) sketch the graph of $y = f(x)$,   [2]

    (b) state, with a reason, whether f has an inverse.   [1]

10 A function f is defined by $f : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant \pi$.

   (i) Find the range of f.   N~ 16—12 - 10   [2]

   (ii) Sketch the graph of $y = f(x)$.   [2]

   (iii) Solve the equation $f(x) = 6$, giving answers in terms of π.   [3]

The function g is defined by $g : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant k$, where k is a constant.

   (iv) State the largest value of k for which g has an inverse.   [1]

   (v) For this value of k, find an expression for $g^{-1}(x)$.   [3]

11 The function $f : x \mapsto 6 - 4\cos(\tfrac{1}{2}x)$ is defined for $0 \leqslant x \leqslant 2\pi$.

    (i) Find the exact value of x for which $f(x) = 4$.   [3]

    (ii) State the range of f.   [2]

    (iii) Sketch the graph of $y = f(x)$.   [2]

    (iv) Find an expression for $f^{-1}(x)$.   [3]

N-14-12-11
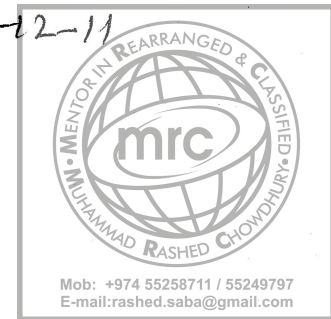
1 2   A function f is defined by $f : x \mapsto 3 - 2\tan(\frac{1}{2}x)$ for $0 \leqslant x < \pi$.   N-10-11-7

   (i) State the range of f.   [1]

   (ii) State the exact value of $f(\frac{2}{3}\pi)$.   [1]

   (iii) Sketch the graph of $y = f(x)$.   [2]

   (iv) Obtain an expression, in terms of x, for $f^{-1}(x)$.   [3]
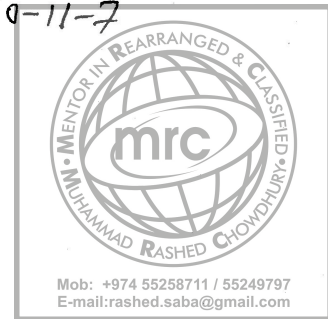
13 A function f is defined by $f : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant 2\pi$.

    (i) Solve the equation $f(x) = 0$.   [3]

    (ii) Find the range of f.   [2]

    (iii) Sketch the graph of $y = f(x)$.   [2]

A function g is defined by $g : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant k$.

    (iv) State the maximum value of **k** for which g has an inverse.   [1]

    (v) Obtain an expression for $g^{-1}(x)$.   [2]
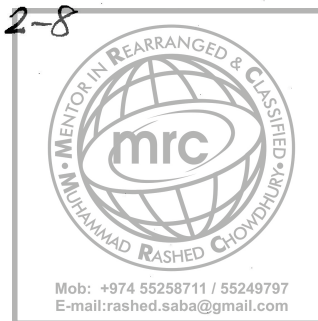
14 The function f is defined by

$$f : x \mapsto 3x - 2 \quad \text{for } x \in \mathbb{R}.$$

N-8-10

(i) Sketch, in a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the two graphs. [2]

The function g is defined by

$$g : x \mapsto 6x - x^2 \quad \text{for } x \in \mathbb{R}.$$

(ii) Express $gf(x)$ in terms of x, and hence show that the maximum value of $gf(x)$ is 9. [5]

The function h is defined by

$$h : x \mapsto 6x - x^2 \quad \text{for } x \geqslant 3.$$
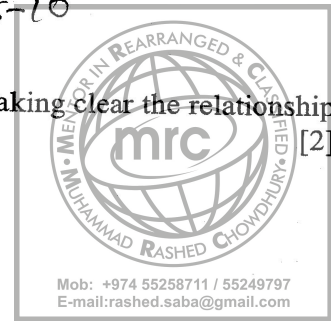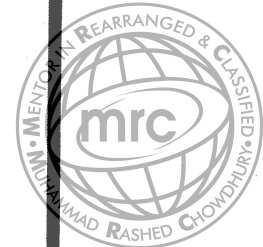
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Pure Mathematics-1
TOPIC- FUNCTIONS
SUB-TOPIC- Composite

1 Functions f and g are defined by

$$f : x \mapsto 3x + 2, \quad x \in \mathbb{R},$$
$$g : x \mapsto 4x - 12, \quad x \in \mathbb{R}.$$

Solve the equation $f^{-1}(x) = gf(x)$. [4]

2 The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 3x + a,$$
$$g : x \mapsto b - 2x,$$

where a and b are constants. Given that $ff(2) = 10$ and $g^{-1}(2) = 3$, find

  (i) the values of a and b, [4]

  (ii) an expression for $fg(x)$. [2]
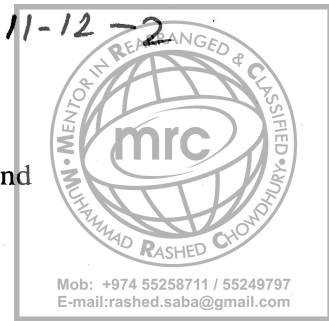
3 The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 4x - 2x^2,$$
$$g : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots. [3]
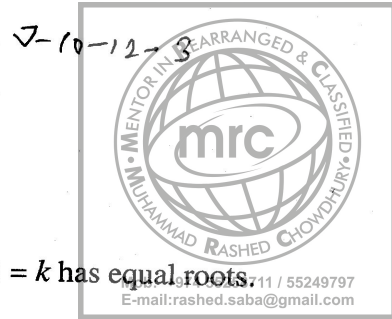
04   Functions f and g are defined by

$$f : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R}, \ x \neq \frac{3}{2}.$$

Solve the equation $ff(x) = gf(2)$.

[3]
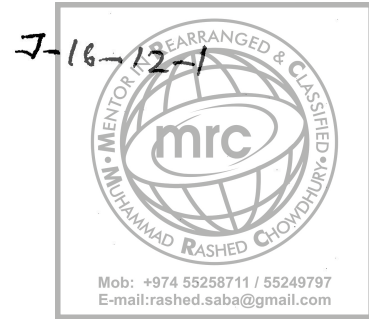
05 The functions f and g are defined as follows:

$$f : x \mapsto x^2 - 2x, \quad x \in \mathbb{R},$$
$$g : x \mapsto 2x + 3, \quad x \in \mathbb{R}.$$

J-4-10

(i) Find the set of values of x for which $f(x) > 15$. [3]

(ii) Find the range of f and state, with a reason, whether f has an inverse. [4]

(iii) Show that the equation $gf(x) = 0$ has no real solutions. [3]

(iv) Sketch, in a single diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$, making clear the relationship between the graphs. [2]
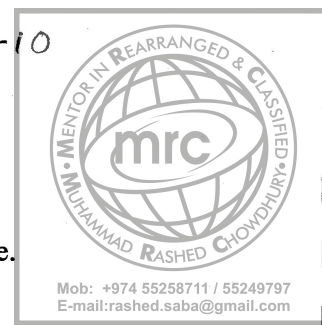
6 The functions f and g are defined for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$ by

$$f(x) = \tfrac{1}{2}x + \tfrac{1}{6}\pi,$$
$$g(x) = \cos x.$$

Solve the following equations for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) $gf(x) = 1$, giving your answer in terms of π. [2]

(ii) $fg(x) = 1$, giving your answers correct to 2 decimal places. [4]
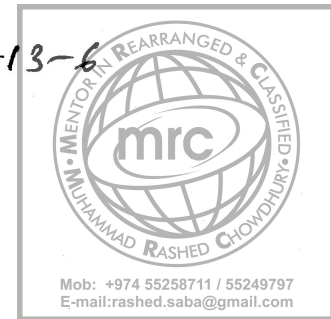
0 2 The functions f and g are defined by

$$f(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$

$$g(x) = \frac{4}{5x + 2} \quad \text{for } x \geq 0.$$

(i) Find and simplify an expression for fg(x) and state the range of fg. [3]

(ii) Find an expression for $g^{-1}(x)$ and find the domain of g^{-1}. [5]

N-16-11-8

8 Functions f and g are defined by

$$f : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$g : x \mapsto \frac{9}{2-x} \quad \text{for } x \in \mathbb{R}, \, x \neq 2.$$

(i) Find the values of k for which the equation $fg(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $fg(x) = x$ for the values of k found in part (i). [3]

9   Functions f and g are defined by

$$f : x \mapsto 2x + 3 \quad \text{for } x \leqslant 0,$$
$$g : x \mapsto x^2 - 6x \quad \text{for } x \leqslant 3.$$

(i) Express $f^{-1}(x)$ in terms of x and solve the equation $f(x) = f^{-1}(x)$. [3]

(ii) On the same diagram sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing the coordinates of their point of intersection and the relationship between the graphs. [3]

(iii) Find the set of values of x which satisfy $gf(x) \leqslant 16$. [5]
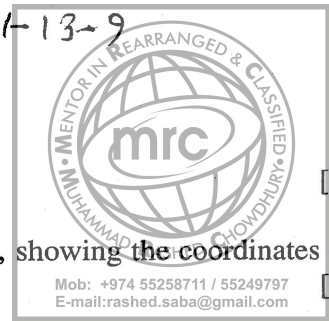
10 Functions f and g are defined by

$$f : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$g : x \mapsto 2(x - 1)^3 + 8, \quad x > 1.$$

(i) Evaluate fg(2). [2]

(ii) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $g'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [4]

11 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 1,$$
$$g : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for fg(x) and gf(x). [2]

(ii) Hence find the value of a for which fg(a) = gf(a). [3]

(iii) Find the value of b ($b \neq a$) for which g(b) = b. [2]

(iv) Find and simplify an expression for $f^{-1}g(x)$. [2]

The function h is defined by

$$h : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $h^{-1}(x)$. [2]

FUNCTIONS

12 Functions f and g are defined by

$$f : x \mapsto 2x + 5 \quad \text{for } x \in \mathbb{R},$$

$$g : x \mapsto \frac{8}{x - 3} \quad \text{for } x \in \mathbb{R},\ x \neq 3.$$
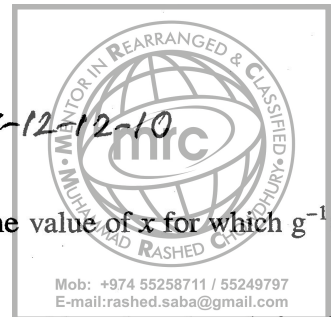
(i) Obtain expressions, in terms of x, for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined. [4]

(ii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs. [3]

(iii) Given that the equation $fg(x) = 5 - kx$, where k is a constant, has no solutions, find the set of possible values of k. [5]

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1 3 Functions f and g are defined by

$$f : x \mapsto 2x - 3, \quad x \in \mathbb{R},$$
$$g : x \mapsto x^2 + 4x, \quad x \in \mathbb{R}.$$

J— 14—13—10

(i) Solve the equation ff$(x) = 11$. [2]

(ii) Find the range of g. [2]

(iii) Find the set of values of x for which g$(x) > 12$. [3]

(iv) Find the value of the constant p for which the equation gf$(x) = p$ has two equal roots. [3]

Function h is defined by h $: x \mapsto x^2 + 4x$ for $x \geqslant k$, and it is given that h has an inverse.

(v) State the smallest possible value of k. [1]

(vi) Find an expression for h$^{-1}(x)$. [4]

FUNCTIONS

14 Functions f and g are defined by

$$f : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$g : x \mapsto x \quad \text{for } 0 \leqslant x \leqslant 10.$$

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) State the domain of f^{-1}. [1]

(iv) Sketch on the same diagram the graphs of $y = f(x)$, $y = g(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [4]

$N \sim 11 - 11 - 11$

(v) Find an expression for $f^{-1}(x)$. [3]
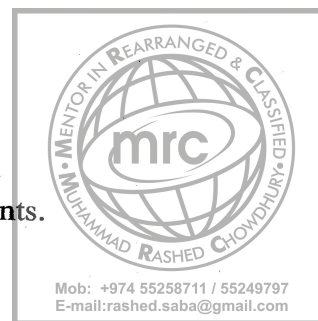
15 Functions f and g are defined by

$$f : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$
$$g : x \mapsto \frac{9}{x+2} \quad \text{for } x \in \mathbb{R}, \ x \neq -2.$$

(i) Find the values of k for which the equation $f(x) = g(x)$ has two equal roots and solve the equation $f(x) = g(x)$ in these cases. [6]

(ii) Solve the equation $fg(x) = 5$ when $k = 6$. [3]

(iii) Express $g^{-1}(x)$ in terms of x. [2]
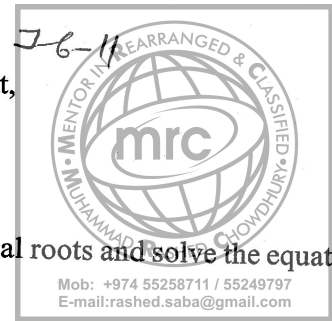
16 Functions f and g are defined by

$$f : x \mapsto 2x + 1, \quad x \in \mathbb{R}, \quad x > 0,$$
$$g : x \mapsto \frac{2x-1}{x+3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$
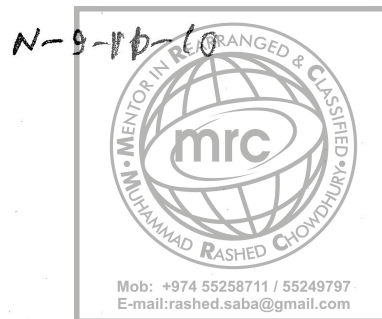
(i) Solve the equation $gf(x) = x$. [3]

(ii) Express $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [4]

(iii) Show that the equation $g^{-1}(x) = x$ has no solutions. [3]

(iv) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]
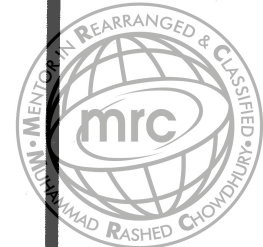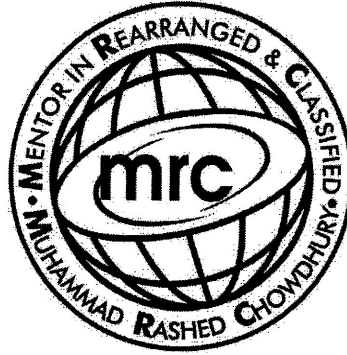
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Pure Mathematics-1

TOPIC- FUNCTIONS

SUB-TOPIC- Sketch

1.
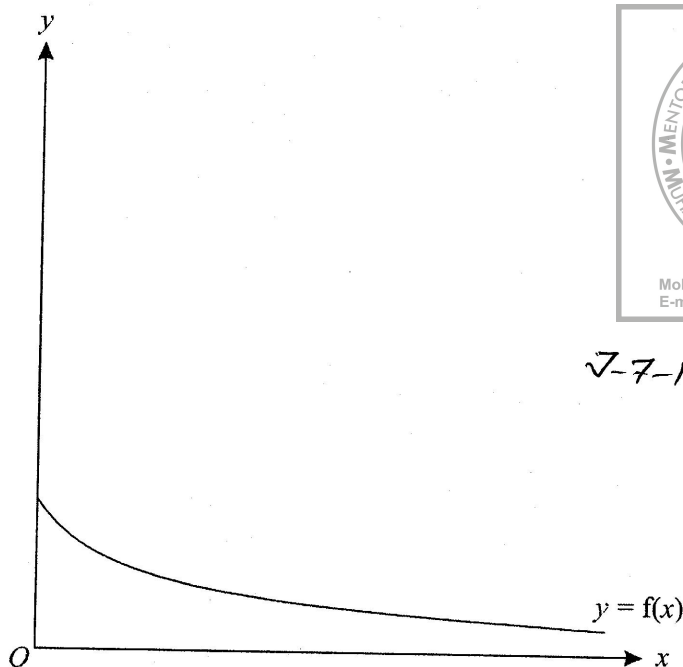
The diagram shows the graph of $y = f(x)$, where $f : x \mapsto \dfrac{6}{2x+3}$ for $x \geqslant 0$.

(i) Find an expression, in terms of x, for $f'(x)$ and explain how your answer shows that f is a decreasing function.

[3]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}.

[4]

(iii) Copy the diagram and, on your copy, sketch the graph of $y = f^{-1}(x)$, making clear the relationship between the graphs.

[2]

The function g is defined by $g : x \mapsto \frac{1}{2}x$ for $x \geqslant 0$.

(iv) Solve the equation $fg(x) = \frac{3}{2}$.

[3]

0 2
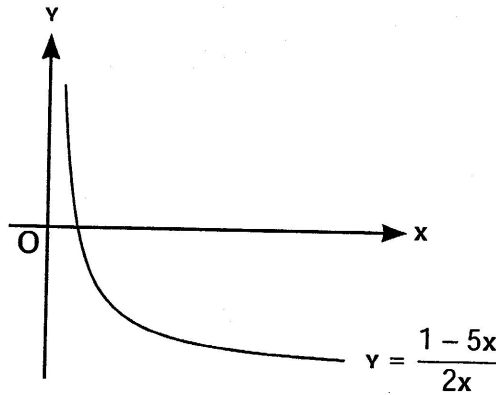
$$Y = \frac{1 - 5x}{2x}$$

J-15-13-6

The diagram shows the graph of $y = f^{-1}(x)$, where f^{-1} is defined by $f^{-1}(x) = \dfrac{1 - 5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $f(x)$ and state the domain of f.   [5]

(ii) The function g is defined by $g(x) = \dfrac{1}{x}$ for $x \geqslant 1$. Find an expression for $f^{-1}g(x)$, giving your answer in the form $ax + b$, where **a** and **b** are constants to be found.   [2]

0 3
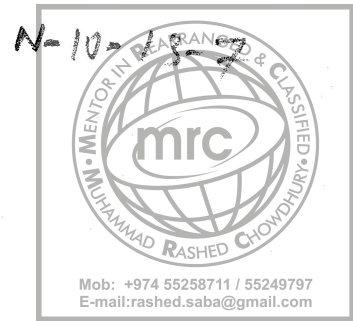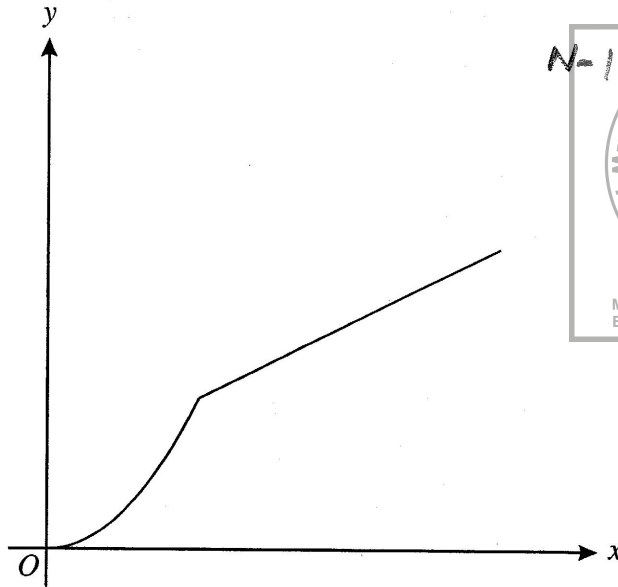
The diagram shows the function f defined for $0 \leqslant x \leqslant 6$ by

$$x \mapsto \tfrac{1}{2}x^2 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$x \mapsto \tfrac{1}{2}x + 1 \quad \text{for } 2 < x \leqslant 6.$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]

(iii) Obtain expressions to define $f^{-1}(x)$, giving the set of values of x for which each expression is valid. [4]

0 4

$y = 11 - x^2$

$A\,(p, q)$

$y = 5 - x$
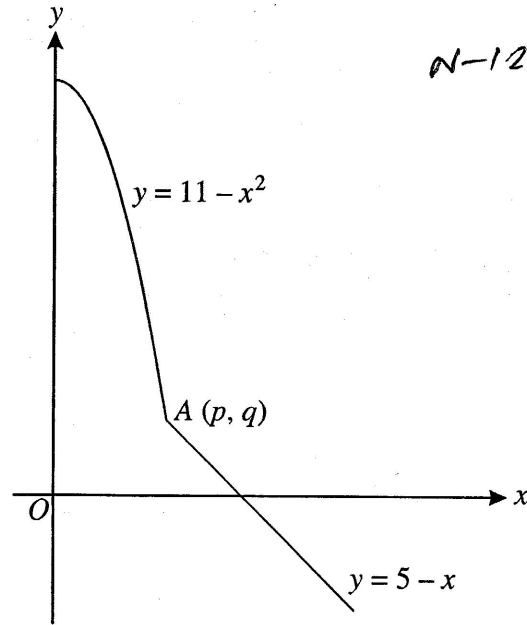
N-12-13-7
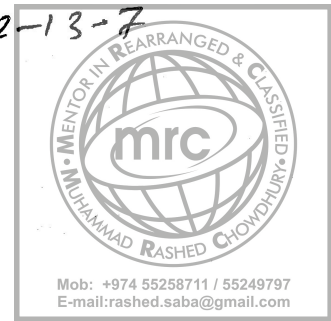
(i) The diagram shows part of the curve $y = 11 - x^2$ and part of the straight line $y = 5 - x$ meeting at the point $A\,(p, q)$, where p and q are positive constants. Find the values of p and q. [3]

(ii) The function f is defined for the domain $x \geqslant 0$ by

$$f(x) = \begin{cases} 11 - x^2 & \text{for } 0 \leqslant x \leqslant p, \\ 5 - x & \text{for } x > p. \end{cases}$$

Express $f^{-1}(x)$ in a similar way. [5]

0 5
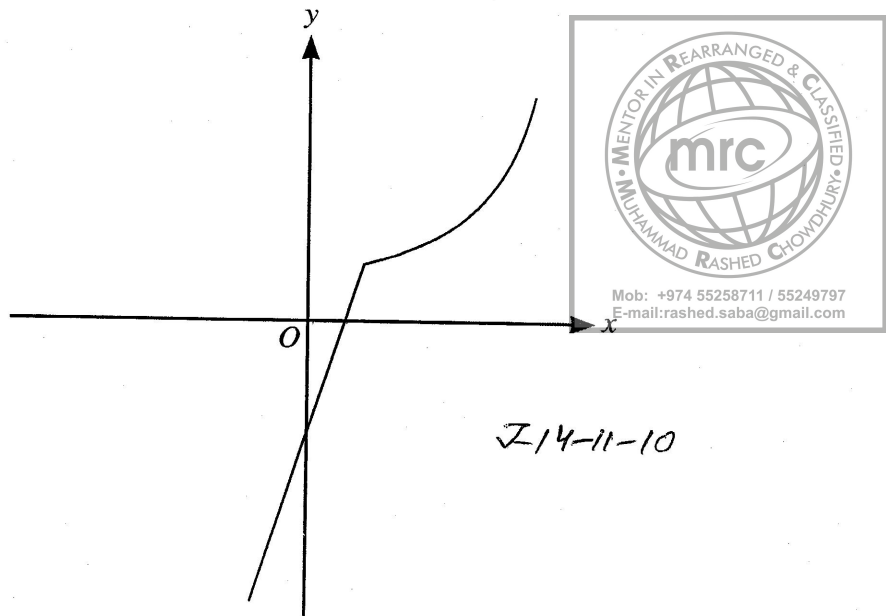
The diagram shows the function f defined for $-1 \leqslant x \leqslant 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leqslant x \leqslant 1, \\ \dfrac{4}{5-x} & \text{for } 1 < x \leqslant 4. \end{cases}$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]

(iii) Obtain expressions to define the function f^{-1}, giving also the set of values for which each expression is valid. [6]
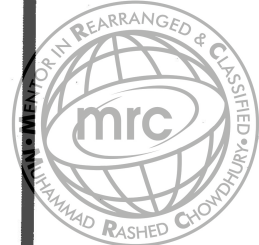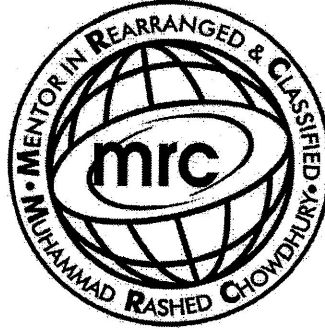
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Pure Mathematics-1

TOPIC- FUNCTIONS

SUB-TOPIC- Test

9 The function f is defined by $f : x \mapsto \dfrac{2}{3 - 2x}$ for $x \in \mathbb{R}$, $x \neq \frac{3}{2}$.

(i) Find an expression for $f^{-1}(x)$. [3]

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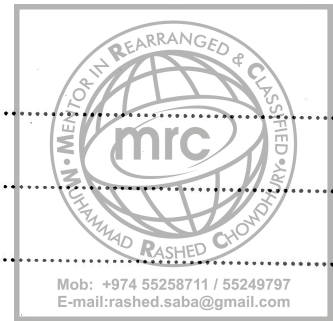
The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

(ii) Find the value of a for which $gf(-1) = 3$. [3]

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(iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

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FUNCTIONS

10 The function f is defined by $f(x) = 3\tan(\frac{1}{2}x) - 2$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

(ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1}. [5]

(iii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]

(iii) Find f(x), and hence find the minimum value of f. J-17-13-11 [5]

9 **(i)** Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $f(x) = 9x^2 - 6x + 6$ for $x \geqslant p$, where p is a constant.

(ii) State the smallest value of p for which f is a one-one function. [1]

(iii) For this value of p, obtain an expression for $f^{-1}(x)$, and state the domain of f^{-1}. [4]

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(iv) State the set of values of q for which the equation $f(x) = q$ has no solution. [1]

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[Turn over