CLASSIFIED

International Examinations Papers

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Probability & Statistics 1

TOPIC- The binomial distribution

The binomial distribution

1 A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive. [3]

5-62-15 B1 P

2 The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers greater than 14. [4]

3 In Restaurant Bijoux 13% of customers rated the food as 'poor', 22% of customers rated the food as 'satisfactory' and 65% rated it as 'good'. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken.

(i) Find the probability that more than 2 and fewer than 12 of them rated the food as 'good'. [3]

On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken.

(ii) Find the smallest value of n for which the probability that at least 1 person will rate the food as 'poor' is greater than 0.95. [3]

The binomial distribution

4 In a certain mountainous region in winter, the probability of more than 20 cm of snow falling on any particular day is 0.21.

(i) Find the probability that, in any 7-day period in winter, fewer than 5 days have more than 20 cm of snow falling. [3]

(ii) For 4 randomly chosen 7-day periods in winter, find the probability that exactly 3 of these periods will have at least 1 day with more than 20 cm of snow falling. [4]

5 An English examination consists of 8 questions in Part A and 3 questions in Part B. Candidates must choose 6 questions. The order in which questions are chosen does not matter. Find the number of ways in which the 6 questions can be chosen in each of the following cases.

(i) There are no restrictions on which questions can be chosen. [1]

(ii) Candidates must choose at least 4 questions from Part A. [3]

(iii) Candidates must either choose both question 1 and question 2 in Part A, or choose neither of these questions. [3]

6 A box of biscuits contains 30 biscuits, some of which are wrapped in gold foil and some of which are unwrapped. Some of the biscuits are chocolate-covered. 12 biscuits are wrapped in gold foil, and of these biscuits, 7 are chocolate-covered. There are 17 chocolate-covered biscuits in total. S-62-12

(i) Copy and complete the table below to show the number of biscuits in each category. [2]

	Wrapped in gold foil	Unwrapped	Total
Chocolate-covered			
Not chocolate-covered			
Total			30

A biscuit is selected at random from the box.

(ii) Find the probability that the biscuit is wrapped in gold foil. [1]

The biscuit is returned to the box. An unwrapped biscuit is then selected at random from the box.

(iii) Find the probability that the biscuit is chocolate-covered. [1]

The biscuit is returned to the box. A biscuit is then selected at random from the box.

(iv) Find the probability that the biscuit is unwrapped, given that it is chocolate-covered. [1]

The biscuit is returned to the box. Nasir then takes 4 biscuits without replacement from the box.

(v) Find the probability that he takes exactly 2 wrapped biscuits. [4]

7 Dayo chooses two digits at random, without replacement, from the 9-digit number 113 333 555.

(i) Find the probability that the two digits chosen are equal. [3]

(ii) Find the probability that one digit is a 5 and one digit is not a 5. [3]

(iii) Find the probability that the first digit Dayo chose was a 5, given that the second digit he chose is not a 5. [4]

(iv) The random variable X is the number of 5s that Dayo chooses. Draw up a table to show the probability distribution of X. [3]

The binomial distribution

S-62-11

1 A biased die was thrown 20 times and the number of 5s was noted. This experiment was repeated many times and the average number of 5s was found to be 4.8. Find the probability that in the next 20 throws the number of 5s will be less than three. [4]

P 13

The binomial distribution

1 In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

N/6/o/15 B

The binomial distribution

1 Biscuits are sold in packets of 18. There is a constant probability that any biscuit is broken, independently of other biscuits. The mean number of broken biscuits in a packet has been found to be 2.7. Find the probability that a packet contains between 2 and 4 (inclusive) broken biscuits. [4]

S-61-11

6 (i) State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

(ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]

(iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

The binomial distribution

4 When people visit a certain large shop, on average 34% of them do not buy anything, 53% spend less than $50 and 13% spend at least $50.

(i) 15 people visiting the shop are chosen at random. Calculate the probability that at least 14 of them buy something. [3]

(ii) n people visiting the shop are chosen at random. The probability that none of them spends at least $50 is less than 0.04. Find the smallest possible value of n. [3]

5 Screws are sold in packets of 15. Faulty screws occur randomly. A large number of packets are tested for faulty screws and the mean number of faulty screws per packet is found to be 1.2.

N-61-14

(i) Show that the variance of the number of faulty screws in a packet is 1.104. [2]

(ii) Find the probability that a packet contains at most 2 faulty screws. [3]

Damien buys 8 packets of screws at random.

(iii) Find the probability that there are exactly 7 packets in which there is at least 1 faulty screw. [4]

4 When people visit a certain large shop, on average 34% of them do not buy anything, 53% spend less than \$50 and 13% spend at least \$50. S-62-16

(i) 15 people visiting the shop are chosen at random. Calculate the probability that at least 14 of them buy something. [3]

(ii) n people visiting the shop are chosen at random. The probability that none of them spends at least \$50 is less than 0.04. Find the smallest possible value of n. [3]

2 A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

7 **(i)** In a certain country, the daily minimum temperature, in °C, in winter has the distribution N(8, 24). Find the probability that a randomly chosen winter day in this country has a minimum temperature between 7 °C and 12 °C. [3]

The daily minimum temperature, in °C, in another country in winter has a normal distribution with mean μ and standard deviation 2μ.

(ii) Find the proportion of winter days on which the minimum temperature is below zero. [2]

(iii) 70 winter days are chosen at random. Find how many of these would be expected to have a minimum temperature which is more than three times the mean. [3]

(iv) The probability of the minimum temperature being above 6 °C on any winter day is 0.0735. Find the value of μ. [3]

3 A team of 4 is to be randomly chosen from 3 boys and 5 girls. The random variable X is the number of girls in the team.

N-61-11

(i) Draw up a probability distribution table for X. [4]

(ii) Given that $\mathrm{E}(X) = \frac{5}{2}$, calculate $\mathrm{Var}(X)$. [2]

3 Christa takes her dog for a walk every day. The probability that they go to the park on any day is 0.6. If they go to the park there is a probability of 0.35 that the dog will bark. If they do not go to the park there is a probability of 0.75 that the dog will bark.

(i) Find the probability that they go to the park on more than 5 of the next 7 days. [2]

(ii) Find the probability that the dog barks on any particular day. [2]

(iii) Find the variance of the number of times they go to the park in 30 days. [1]

5 A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

S-62-15

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S. [5]

7 Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) $= \frac{1}{2}$. [3]

(ii) Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Event R is 'the sum of the numbers on the three cards is 11'. Event S is 'the number on the first card taken is a 3'.

(iii) Determine whether events R and S are independent. Justify your answer. [3]

(iv) Determine whether events R and S are exclusive. Justify your answer. [1]

3 In a large consignment of mangoes, 15% of mangoes are classified as small, 70% as medium and 15% as large.

N-63-13 P B

(i) Yue-chen picks 14 mangoes at random. Find the probability that fewer than 12 of them are medium or large. [3]

(ii) Yue-chen picks n mangoes at random. The probability that none of these n mangoes is small is at least 0.1. Find the largest possible value of n. [3]

5 Fiona uses her calculator to produce 12 random integers between 7 and 21 inclusive. The random variable X is the number of these 12 integers which are multiples of 5.

5-61-13

(i) State the distribution of X and give its parameters. [3]

(ii) Calculate the probability that X is between 3 and 5 inclusive. [3]

Fiona now produces n random integers between 7 and 21 inclusive.

(iii) Find the least possible value of n if the probability that none of these integers is a multiple of 5 is less than 0.01. [3]

3 The number of books read by members of a book club each year has the binomial distribution B(12, 0.7).

N-63-14

(i) State the greatest number of books that could be read by a member of the book club in a particular year and find the probability that a member reads this number of books. [2]

(ii) Find the probability that a member reads fewer than 10 books in a particular year. [3]

5 A company set up a display consisting of 20 fireworks. For each firework, the probability that it fails to work is 0.05, independently of other fireworks.

N-61-12

(i) Find the probability that more than 1 firework fails to work. [3]

The 20 fireworks cost the company $24 each. 450 people pay the company $10 each to watch the display. If more than 1 firework fails to work they get their money back.

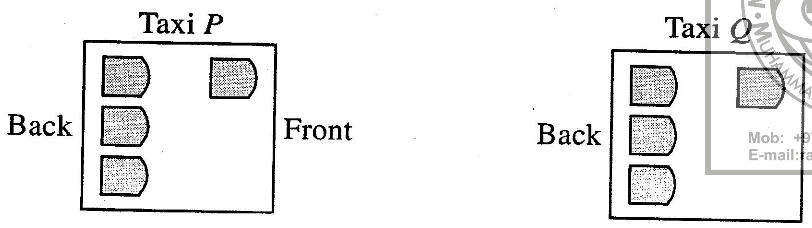
(ii) Calculate the expected profit for the company. [4]

3 (i) Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown. Find the probability that the numbers shown on the four dice add up to 5. N-62-14 [3]

(ii) Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown on 7 occasions. Find the probability that the numbers shown on the four dice add up to 5 on exactly 1 or 2 of the 7 occasions. [4]

4 A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(i) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi. Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

2 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X. S-61-16

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

6 A fair tetrahedral die has four triangular faces, numbered 1, 2, 3 and 4. The score when this die is thrown is the number on the face that the die lands on. This die is thrown three times. The random variable X is the sum of the three scores.

N-62-12

(i) Show that $P(X = 9) = \frac{10}{64}$. [3]

(ii) Copy and complete the probability distribution table for X. [3]

x	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{64}$	$\frac{3}{64}$			$\frac{12}{64}$					

P
K

(iii) Event R is 'the sum of the three scores is 9'. Event S is 'the product of the three scores is 16'. Determine whether events R and S are independent, showing your working. [5]