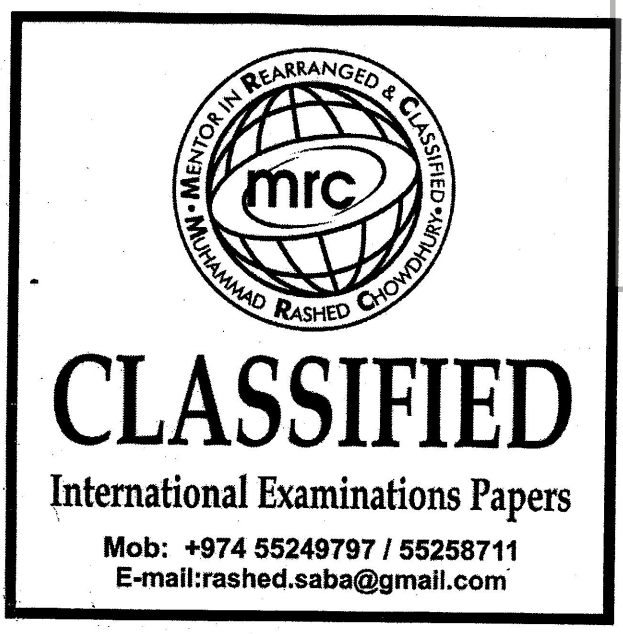


Probability & Statistics 1

TOPIC- The normal distribution

mean

The normal distribution

1 The random variable Y is normally distributed with mean equal to five times the standard deviation. It is given that $P(Y > 20) = 0.0732$. Find the mean. [3]

5-62-13

0 2 The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ. [3]

5 The random variable X is normally distributed with mean μ and standard deviation $\frac{1}{4}\mu$. It is given that $P(X > 20) = 0.04$.

(i) Find μ. [3]

(ii) Find $P(10 < X < 20)$. [3]

(iii) 250 independent observations of X are taken. Find the probability that at least 235 of them are less than 20. [5]

0 3 The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ. [4]

5-G1-15

4 Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

N-61-16

(i) Find the probability that a randomly chosen packet weighs less than 1 kg. [3]

(ii) How many packets of rice, on average, would the machine fill from 1000 kg of rice? [1]

The factory manager wants to produce more packets of rice. He changes the settings on the machine so that the standard deviation is the same but the mean is reduced to μ kg. With this mean the probability that a packet weighs less than 1 kg is 0.0388.

(iii) Find the value of μ. [3]

(iv) How many packets of rice, on average, would the machine now fill from 1000 kg of rice? [1]

N-61-16

6 The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

(i) Find the value of μ. [3]

(ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm. [6]

The normal distribution

1 In a normal distribution with mean 9.3, the probability of a randomly chosen value being greater than 5.6 is 0.85. Find the standard deviation. [3]

N-63-12

0 2 The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

The normal distribution

0 3 The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

4 The random variable X is normally distributed with mean μ and standard deviation σ.

(i) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. [3]

(ii) With a different relationship between μ and σ, it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ. [3]

The normal distribution

5 The heights of school desks have a normal distribution with mean 69 cm and standard deviation σ cm. It is known that 15.5% of these desks have a height greater than 70 cm.

5-63-16

2

(i) Find the value of σ. [3]

When Jodu sits at a desk, his knees are at a height of 58 cm above the floor. A desk is comfortable for Jodu if his knees are at least 9 cm below the top of the desk. Jodu's school has 300 desks.

(ii) Calculate an estimate of the number of these desks that are comfortable for Jodu. [5]

0 6 Packets of tea are labelled as containing 250 g. The actual weight of tea in a packet has a normal distribution with mean 260 g and standard deviation σ g. Any packet with a weight less than 250 g is classed as 'underweight'. Given that 1% of packets of tea are underweight, find the value of σ. [3]

N-03-14

The normal distribution

07 The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

(i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ. [3]

(ii) 170 seeds are sown. Find the expected number of seeds which take between 131 and 141 hours to germinate. [4]

The normal distribution

N-61-12

0 8 Lengths of rolls of parcel tape have a normal distribution with mean 75 m, and 15% of the rolls have lengths less than 73 m.

(i) Find the standard deviation of the lengths. [3]

Alison buys 8 rolls of parcel tape.

(ii) Find the probability that fewer than 3 of these rolls have lengths more than 77 m. [3]

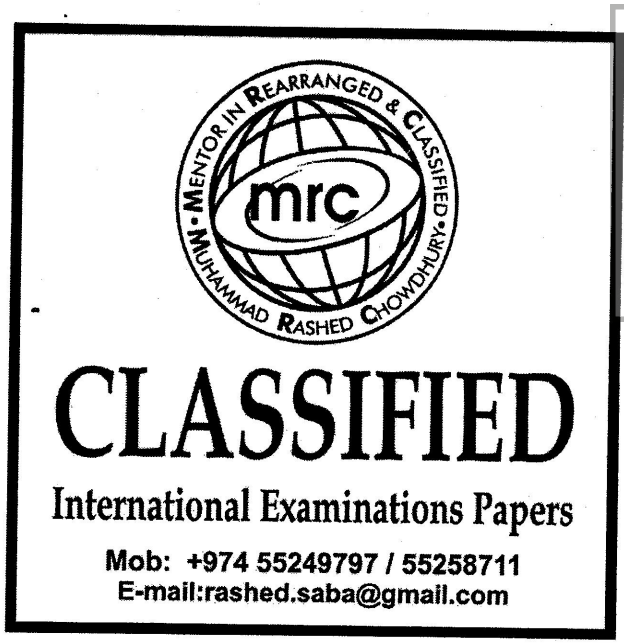
The normal distribution

09 The amount of fibre in a packet of a certain brand of cereal is normally distributed with mean 160 grams. 19% of packets of cereal contain more than 190 grams of fibre.

N- 62-13 2

(i) Find the standard deviation of the amount of fibre in a packet. [3]

(ii) Kate buys 12 packets of cereal. Find the probability that at least 1 of the packets contains more than 190 grams of fibre. [2]



Probability & Statistics 1

TOPIC- The normal distribution

μ and σ

The normal distribution

1 The random variable X is normally distributed and is such that the mean μ is three times the standard deviation σ. It is given that $P(X < 25) = 0.648$.

N-63-11 Z

(i) Find the values of μ and σ. [4]

(ii) Find the probability that, from 6 random values of X, exactly 4 are greater than 25. [2]

2 The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ. [4]

N-81-15

2

(BL 62-12)

03 **(a)** The random variable X is normally distributed with mean 82 and standard deviation 7.4. Find the value of q such that $P(82 - q < X < 82 + q) = 0.44$. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. It is given that $5\mu = 2\sigma^2$ and that $P(Y < \frac{1}{2}\mu) = 0.281$. Find the values of μ and σ. [4]

The normal distribution

0 4 **(a)** Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.

(i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]

(ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t. [3]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ. [5]

5 **(a)** The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

(b) It is given that $Y \sim N(33, 21)$. Find the value of a given that $P(33 - a < Y < 33 + a) = 0.5$. [4]

The normal distribution

0 6 The mean of a certain normally distributed variable is four times the standard deviation. The probability that a randomly chosen value is greater than 5 is 0.15.

(i) Find the mean and standard deviation. [4]

(ii) 200 values of the variable are chosen at random. Find the probability that at least 160 of these values are less than 5. [5]

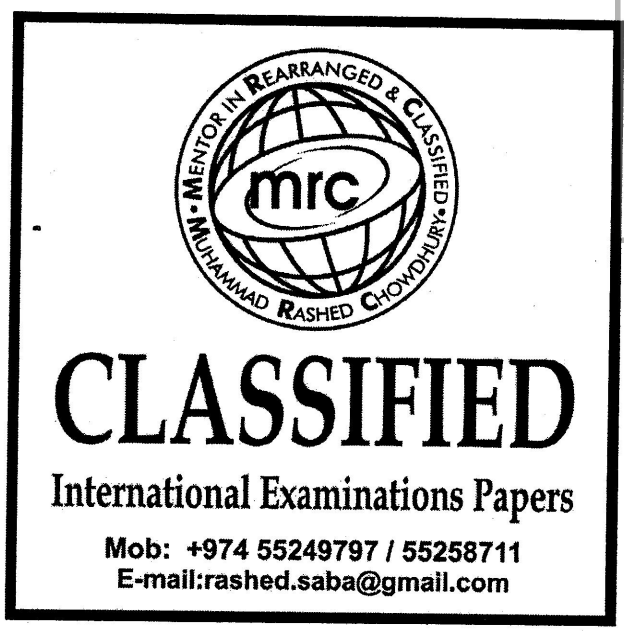
The normal distribution

7 The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.

(i) Find the mean and standard deviation of the heights the children can jump. [5]

(ii) Find the probability that a randomly chosen child will **not** be able to jump a height of 145 cm. [3]

(iii) Find the probability that, of 8 randomly chosen children, at least 2 will be able to jump a height of more than 135 cm. [3]



Probability & Statistics 1

TOPIC- The normal distribution

Finding constant

The normal distribution

1 It is given that $X \sim N(30, 49)$, $Y \sim N(30, 16)$ and $Z \sim N(50, 16)$. On a single diagram, with the horizontal axis going from 0 to 70, sketch three curves to represent the distributions of X, Y and Z. [3]

The normal distribution

1 It is given that $X \sim N(1.5, 3.2^2)$. Find the probability that a randomly chosen value of X is less than -2.4. [3]

N-62-13

The normal distribution

1 It is given that $X \sim N(28.3, 4.5)$. Find the probability that a randomly chosen value of X lies between 25 and 30. [3]

6 The time in minutes taken by Peter to walk to the shop and buy a newspaper is normally distributed with mean 9.5 and standard deviation 1.3.

5-62-16

(i) Find the probability that on a randomly chosen day Peter takes longer than 10.2 minutes. [3]

(ii) On 90% of days he takes longer than t minutes. Find the value of t. [3]

(iii) Calculate an estimate of the number of days in a year (365 days) on which Peter takes less than 8.8 minutes to walk to the shop and buy a newspaper. [3]

3 Cans of lemon juice are supposed to contain 440 ml of juice. It is found that the actual volume of juice in a can is normally distributed with mean 445 ml and standard deviation 3.6 ml.

(i) Find the probability that a randomly chosen can contains less than 440 ml of juice. [3]

It is found that 94% of the cans contain between $(445 - c)$ ml and $(445 + c)$ ml of juice.

(ii) Find the value of c. [3]

The normal distribution

1 The random variable X is such that $X \sim N(20, 49)$. Given that $P(X > k) = 0.25$, find the value of k. [3]

5 Gem stones from a certain mine have weights, X grams, which are normally distributed with mean 1.9 g and standard deviation 0.55 g. These gem stones are sorted into three categories for sale depending on their weights, as follows.

Small: under 1.2 g Medium: between 1.2 g and 2.5 g Large: over 2.5 g

(i) Find the proportion of gem stones in each of these three categories. [5]

(ii) Find the value of k such that $P(k < X < 2.5) = 0.8$. [4]

6 The time in minutes taken by Peter to walk to the shop and buy a newspaper is normally distributed with mean 9.5 and standard deviation 1.3.

S-62-16

(i) Find the probability that on a randomly chosen day Peter takes longer than 10.2 minutes. [3]

(ii) On 90% of days he takes longer than t minutes. Find the value of t. [3]

(iii) Calculate an estimate of the number of days in a year (365 days) on which Peter takes less than 8.8 minutes to walk to the shop and buy a newspaper. [3]

The normal distribution

5 **(a)** The time, X hours, for which people sleep in one night has a normal distribution with mean 7.15 hours and standard deviation 0.88 hours.

(i) Find the probability that a randomly chosen person sleeps for less than 8 hours in a night. [2]

(ii) Find the value of q such that $P(X < q) = 0.75$. [3]

(b) The random variable Y has the distribution $N(\mu, \sigma^2)$, where $2\sigma = 3\mu$ and $\mu \neq 0$. Find $P(Y > 4\mu)$. [3]

4 In a certain country, on average one student in five has blue eyes.

5-63-13

(i) For a random selection of n students, the probability that none of the students has blue eyes is less than 0.001. Find the least possible value of n. [3]

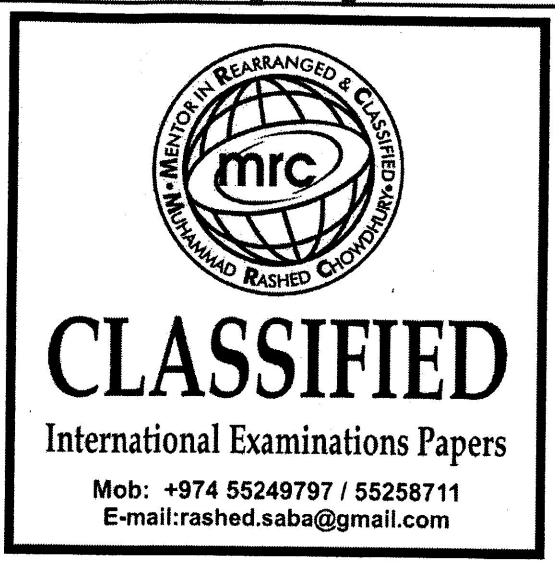
(ii) For a random selection of 120 students, find the probability that fewer than 33 have blue eyes. [4]

5 The random variable X is such that $X \sim N(82, 126)$.

(i) A value of X is chosen at random and rounded to the nearest whole number. Find the probability that this whole number is 84. [3]

(ii) Five independent observations of X are taken. Find the probability that at most one of them is greater than 87. [4]

(iii) Find the value of k such that $P(87 < X < k) = 0.3$. [5]



Probability & Statistics 1

TOPIC- The normal distribution

The normal distribution

1 When a butternut squash seed is sown the probability that it will germinate is 0.86, independently of any other seeds. A market gardener sows 250 of these seeds. Use a suitable approximation to find the probability that more than 210 germinate. [5]

2 On average, 2 apples out of 15 are classified as being underweight. Find the probability that in a random sample of 200 apples, the number of apples which are underweight is more than 21 and less than 35. [5]

3 On a production line making cameras, the probability of a randomly chosen camera being substandard is 0.072. A random sample of 300 cameras is checked. Find the probability that there are fewer than 18 cameras which are substandard. [5]

S-13-15 Z

2 A factory produces flower pots. The base diameters have a normal distribution with mean 14 cm and standard deviation 0.52 cm. Find the probability that the base diameters of exactly 8 out of 10 randomly chosen flower pots are between 13.6 cm and 14.8 cm. [5]

N-62-13

5 On trains in the morning rush hour, each person is either a student with probability 0.36, or an office worker with probability 0.22, or a shop assistant with probability 0.29 or none of these.

(i) 8 people on a morning rush hour train are chosen at random. Find the probability that between 4 and 6 inclusive are office workers. [3]

(ii) 300 people on a morning rush hour train are chosen at random. Find the probability that between 31 and 49 inclusive are neither students nor office workers nor shop assistants. [6]

5 Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

7 In Marumbo, three quarters of the adults own a cell phone.

(i) A random sample of 8 adults from Marumbo is taken. Find the probability that the number of adults who own a cell phone is between 4 and 6 inclusive. [3]

(ii) A random sample of 160 adults from Marumbo is taken. Use an approximation to find the probability that more than 114 of them own a cell phone. [5]

(iii) Justify the use of your approximation in part (ii). [1]

6 The lengths of body feathers of a particular species of bird are modelled by a normal distribution. A researcher measures the lengths of a random sample of 600 body feathers from birds of this species and finds that 63 are less than 6 cm long and 155 are more than 12 cm long.

(i) Find estimates of the mean and standard deviation of the lengths of body feathers of birds of this species. [5]

(ii) In a random sample of 1000 body feathers from birds of this species, how many would the researcher expect to find with lengths more than 1 standard deviation from the mean? [4]

2 Assume that, for a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday. Find the probability that, out of 350 randomly chosen people, at least 47 will have their next birthday on a Monday. [5]

2 The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.

(i) Find the probability that a pencil chosen at random has a length greater than 10.9 cm. [2]

(ii) Find the probability that, in a random sample of 6 pencils, at least two have lengths less than 10.9 cm. [3]

2 When visiting the dentist the probability of waiting less than 5 minutes is 0.16, and the probability of waiting less than 10 minutes is 0.88.

(i) Find the probability of waiting between 5 and 10 minutes. [1]

A random sample of 180 people who visit the dentist is chosen.

(ii) Use a suitable approximation to find the probability that more than 115 of these people wait between 5 and 10 minutes. [5]

7 The times taken to play Beethoven's Sixth Symphony can be assumed to have a normal distribution with mean 41.1 minutes and standard deviation 3.4 minutes. Three occasions on which this symphony is played are chosen at random.

(i) Find the probability that the symphony takes longer than 42 minutes to play on exactly 1 of these occasions. [4]

The times taken to play Beethoven's Fifth Symphony can also be assumed to have a normal distribution. The probability that the time is less than 26.5 minutes is 0.1, and the probability that the time is more than 34.6 minutes is 0.05.

(ii) Find the mean and standard deviation of the times to play this symphony. [5]

(iii) Assuming that the times to play the two symphonies are independent of each other, find the probability that, when both symphonies are played, both of the times are less than 34.6 minutes. [4]

The normal distribution

7 Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

(i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]

(ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

(iii) Find the probability that, of a random sample of 250 full minibuses travelling to Picton, more than 54 will contain exactly 7 passengers carrying backpacks. [6]

6 Ana meets her friends once every day. For each day the probability that she is early is 0.05 and the probability that she is late is 0.75. Otherwise she is on time.

(i) Find the probability that she is on time on fewer than 20 of the next 96 days. [5]

(ii) If she is early there is a probability of 0.7 that she will eat a banana. If she is late she does not eat a banana. If she is on time there is a probability of 0.4 that she will eat a banana. Given that for one particular meeting with friends she does not eat a banana, find the probability that she is on time. [4]

The normal distribution

5 The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are filled with 20 litres of fuel and are driven towards a place 367 km away.

N-62-10 2

(i) For each car, find the probability that it runs out of fuel before it has travelled 367 km. [3]

(ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d. [4]

The normal distribution

7 A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

(iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

(iv) Justify the use of your approximation in part **(iii)**. [1]

The normal distribution

2 When visiting the dentist the probability of waiting less than 5 minutes is 0.16, and the probability of waiting less than 10 minutes is 0.88.

(i) Find the probability of waiting between 5 and 10 minutes. [1]

A random sample of 180 people who visit the dentist is chosen.

(ii) Use a suitable approximation to find the probability that more than 115 of these people wait between 5 and 10 minutes. [5]

The normal distribution

5 In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

5-61-10

(i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]

(ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]

(iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

The normal distribution

4 **(a)** The random variable Y is normally distributed with positive mean μ and standard deviation $\frac{1}{2}\mu$. Find the probability that a randomly chosen value of Y is negative. [3]

(b) The weights of bags of rice are normally distributed with mean 2.04 kg and standard deviation σ kg. In a random sample of 8000 such bags, 253 weighed over 2.1 kg. Find the value of σ. [4]

The normal distribution

3 Buildings in a certain city centre are classified by height as tall, medium or short. The heights can be modelled by a normal distribution with mean 50 metres and standard deviation 16 metres. Buildings with a height of more than 70 metres are classified as tall.

$S - 63 = 13$

2

(i) Find the probability that a building chosen at random is classified as tall. [2]

(ii) The rest of the buildings are classified as medium and short in such a way that there are twice as many medium buildings as there are short ones. Find the height below which buildings are classified as short. [5]

5 A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

(i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$. [1]

(ii) The spinner is spun 3 times. Find the probability that it lands on a different coloured side each time. [3]

(iii) The spinner is spun 136 times. Use a suitable approximation to find the probability that it lands on the blue side fewer than 20 times. [5]

4 The time taken to cook an egg by people living in a certain town has a normal distribution with mean 4.2 minutes and standard deviation 0.6 minutes.

(i) Find the probability that a person chosen at random takes between 3.5 and 4.5 minutes to cook an egg. [3]

12% of people take more than t minutes to cook an egg.

(ii) Find the value of t. [3]

(iii) A random sample of n people is taken. Find the smallest possible value of n if the probability that none of these people takes more than t minutes to cook an egg is less than 0.003. [3]

The normal distribution

3 The times taken by students to get up in the morning can be modelled by a normal distribution with mean 26.4 minutes and standard deviation 3.7 minutes.

(i) For a random sample of 350 students, find the number who would be expected to take longer than 20 minutes to get up in the morning. [3]

(ii) 'Very slow' students are students whose time to get up is more than 1.645 standard deviations above the mean. Find the probability that fewer than 3 students from a random sample of 8 students are 'very slow'. [4]

2 The random variable X is the daily profit, in thousands of dollars, made by a company. X is normally distributed with mean 6.4 and standard deviation 5.2.

(i) Find the probability that, on a randomly chosen day, the company makes a profit between \$10 000 and \$12 000. [3]

(ii) Find the probability that the company makes a loss on exactly 1 of the next 4 consecutive days. [4]

7 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

(iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

7 Each day Annabel eats rice, potato or pasta. Independently of each other, the probability that she eats rice is 0.75, the probability that she eats potato is 0.15 and the probability that she eats pasta is 0.1.

N-63-16

(i) Find the probability that, in any week of 7 days, Annabel eats pasta on exactly 2 days. [2]

(ii) Find the probability that, in a period of 5 days, Annabel eats rice on 2 days, potato on 1 day and pasta on 2 days. [3]

(iii) Find the probability that Annabel eats potato on more than 44 days in a year of 365 days. [5]

5 The weights of letters posted by a certain business are normally distributed with mean 20 g. It is found that the weights of 94% of the letters are within 12 g of the mean.

(i) Find the standard deviation of the weights of the letters. [3]

(ii) Find the probability that a randomly chosen letter weighs more than 13 g. [3]

(iii) Find the probability that at least 2 of a random sample of 7 letters have weights which are more than 12 g above the mean. [3]

2 In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

(i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]

(ii) Give a reason why the use of a normal approximation is justified. [1]

6 The weights of bananas in a fruit shop have a normal distribution with mean 150 grams and standard deviation 50 grams. Three sizes of banana are sold.

Small: under 95 grams
Medium: between 95 grams and 205 grams
Large: over 205 grams

(i) Find the proportion of bananas that are small. [3]

(ii) Find the weight exceeded by 10% of bananas. [3]

The prices of bananas are 10 cents for a small banana, 20 cents for a medium banana and 25 cents for a large banana.

(iii) (a) Show that the probability that a randomly chosen banana costs 20 cents is 0.7286. [1]

(b) Calculate the expected total cost of 100 randomly chosen bananas. [3]

The normal distribution

5 Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

5-61-15

6 **(i)** In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

(ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer. [5]

(iii) Justify your use of the approximation in part **(ii)**. [1]

6 There are a large number of students in Luttley College. 60% of the students are boys. Students can choose exactly one of Games, Drama or Music on Friday afternoons. It is found that 75% of the boys choose Games, 10% of the boys choose Drama and the remainder of the boys choose Music. Of the girls, 30% choose Games, 55% choose Drama and the remainder choose Music.

(i) 6 boys are chosen at random. Find the probability that fewer than 3 of them choose Music. [3]

(ii) 5 Drama students are chosen at random. Find the probability that at least 1 of them is a boy. [6]

7 **(a)** A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(ii) Find the value of m. [3]

(iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

7 Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

(i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]

(ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

(iii) Find the probability that, of a random sample of 250 full minibuses travelling to Picton, more than 54 will contain exactly 7 passengers carrying backpacks. [6]

5 Lengths of a certain type of carrot have a normal distribution with mean 14.2 cm and standard deviation 3.6 cm.

(i) 8% of carrots are shorter than c cm. Find the value of c. [3]

(ii) Rebekah picks 7 carrots at random. Find the probability that at least 2 of them have lengths between 15 and 16 cm. [6]

3 On any day at noon, the probabilities that Kersley is asleep or studying are 0.2 and 0.6 respectively.

(i) Find the probability that, in any 7-day period, Kersley is either asleep or studying at noon on at least 6 days. [3]

(ii) Use an approximation to find the probability that, in any period of 100 days, Kersley is asleep at noon on at most 30 days. [5]