1.

$$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(a) Find $\frac{dy}{dx}$. **(2)**

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k. **(2)**

JU-10-3

2.

The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \quad x > 0$

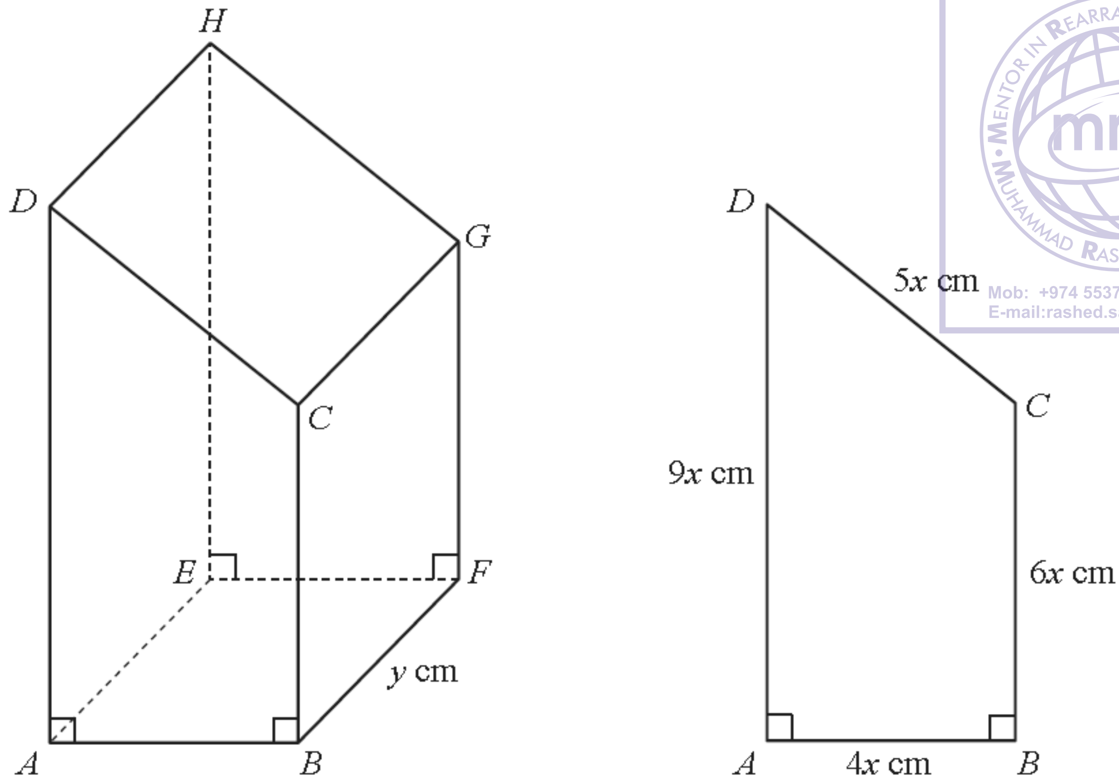
(a) Use calculus to find the coordinates of the turning point on C. **(7)**

(b) Find $\dfrac{d^2y}{dx^2}$. **(2)**

(c) State the nature of the turning point. **(1)**

JA-10-9

3.



$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

14-JA-10

4.

The curve C has equation

$$y = 2\sqrt{x} + \frac{18}{\sqrt{x}} - 1, \qquad x > 0$$

(a) Find

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(5)

(b) Use calculus to find the coordinates of the stationary point of C.

(4)

(c) Determine whether the stationary point is a maximum or minimum, giving a reason for your answer.

(2)

JA-14-3

5.

The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Using the result from part (a), find the coordinates of the turning points of C.

(4)

(c) Find $\dfrac{d^2y}{dx^2}$.

(2)

(d) Hence, or otherwise, determine the nature of the turning points of C.

(2)

JA-6-7

6.

A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)

JA-9-10

7.

Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$.

(4)

JU-5-1

8.

A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)

JA-7-8

9.

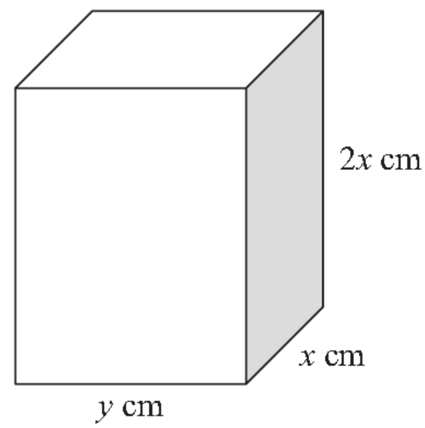


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.
The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.
(5)

(c) Justify that the value of V you have found is a maximum.

(2)

JU-7-10

DIFFERENTIATION-09

10.

Figure 4

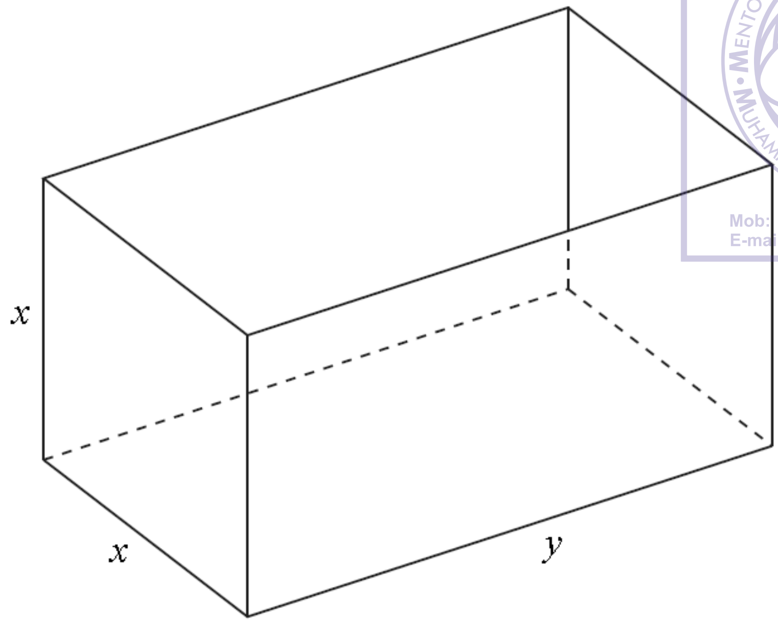


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

JA-8-9

11.

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

JA-11-10

12.

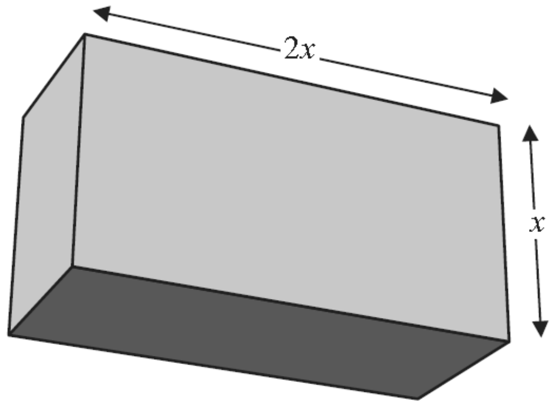


Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

JU-11-8

13.

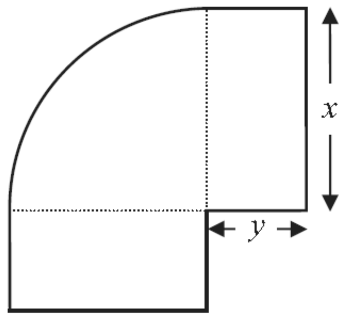


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

JA-12-8

14.

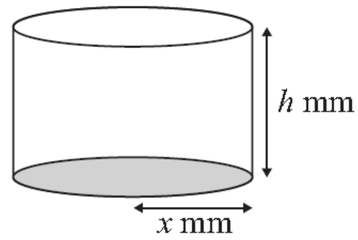


Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x, **(1)**

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$ **(3)**

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum. **(5)**

(d) Calculate the minimum value of A, giving your answer to the nearest integer. **(2)**

(e) Show that this value of A is a minimum. **(2)**

JU-12-8

15.

The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$ **(4)**

(b) Find the x-coordinate of the other turning point Q on the curve. **(1)**

(c) Find $\frac{d^2y}{dx^2}$. **(1)**

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q. **(3)**

JA-13-8

DIFFERENTIATION-09

16.

The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

JU-13-9

17.

A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

JU-15-9

8.