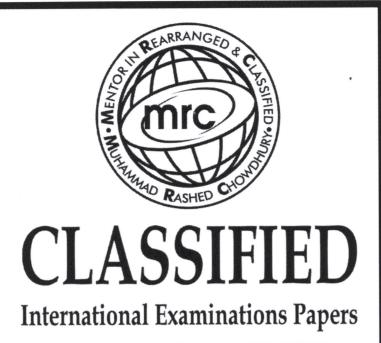
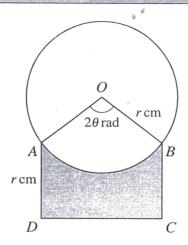
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Pure Mathematics-1 TOPIC- Circular Measure





[3]

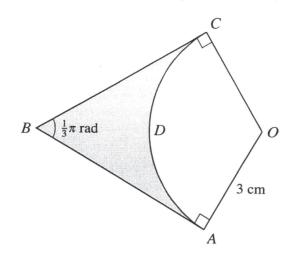
The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and ABCD is a rectangle. Angle $AOB = 2\theta$ radians and AD = r cm.

(i) Express the perimeter of the shaded region in terms of r and θ .

			×
			as record
(ii)	In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shades	d region.	[4]
			, , , * ~
	······		

5-17-12-CM-A.

2





In the diagram, OADC is a sector of a circle with centre O and radius 3 cm. AB and CB are tangents to the circle and angle $ABC = \frac{1}{3}\pi$ radians. Find, giving your answer in terms of $\sqrt{3}$ and π ,

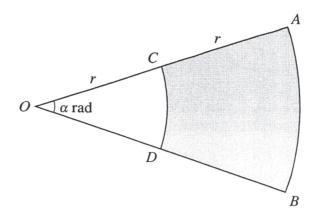
(i) the perimeter of the shaded region. N-14-13-(1)

[3]

(ii) the area of the shaded region.

3

MJ-11-91-K



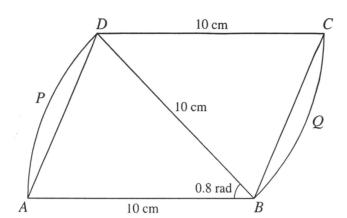


In the diagram OCA and ODB are radii of a circle with centre O and radius 2r cm. Angle $AOB = \alpha$ radians. CD and AB are arcs of circles with centre O and radii r cm and 2r cm respectively. The perimeter of the shaded region ABDC is 4.4r cm.

(i) Find the value of α .

[2]

(ii) It is given that the area of the shaded region is $30 \,\mathrm{cm}^2$. Find the value of r.





In the diagram, ABCD is a parallelogram with AB = BD = DC = 10 cm and angle ABD = 0.8 radians. APD and BQC are arcs of circles with centres B and D respectively.

(i) Find the area of the parallelogram ABCD.

[2]

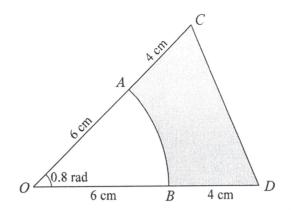
(ii) Find the area of the complete figure ABQCDP.

[2]

(iii) Find the perimeter of the complete figure ABQCDP.

[2]

5





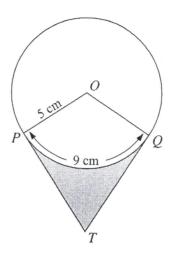
In the diagram, OCD is an isosceles triangle with OC = OD = 10 cm and angle COD = 0.8 radians. The points A and B, on OC and OD respectively, are joined by an arc of a circle with centre O and radius 6 cm. Find

(i) the area of the shaded region,

[3]

(ii) the perimeter of the shaded region.

[4]





In the diagram, the circle has centre O and radius 5 cm. The points P and Q lie on the circle, and the arc length PQ is 9 cm. The tangents to the circle at P and Q meet at the point T. Calculate

(i) angle POQ in radians,

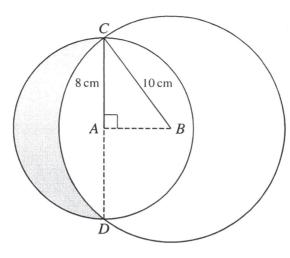
N-8-6) [2]

(ii) the length of PT,

[3]

(iii) the area of the shaded region.

3-17-13-CM



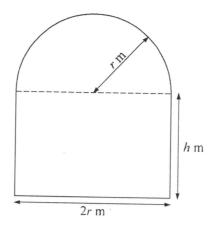


The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD.

(i)	Find angle ABC in radians.	[1]
		••••
		••••
		••••
		••••
		••••
(ii)	Find the area of the shaded region.	[6]
		••••
		••••
		••••
		•••
		•••
		•••

CIRCULAR MEASURE-ARC LENGTH

8





The diagram shows a glass window consisting of a rectangle of height hm and width 2rm and a semicircle of radius rm. The perimeter of the window is 8 m.

(i) Express h in terms of r.

[2]

(ii) Show that the area of the window, $A \,\mathrm{m}^2$, is given by

$$A = 8r - 2r^2 - \frac{1}{2}\pi r^2.$$
 [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value,

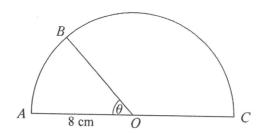
[4]

(iv) determine whether this stationary value is a maximum or a minimum.

[2]

CIRCULAR MEASURE-ARC LENGTH

9

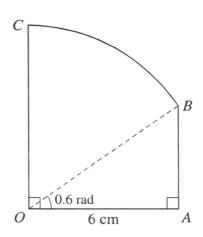




The diagram shows a semicircle ABC with centre O and radius 8 cm. Angle $AOB = \theta$ radians.

(i) In the case where $\theta = 1$, calculate the area of the sector BOC.

- (ii) Find the value of θ for which the perimeter of sector AOB is one half of the perimeter of sector BOC.
- (iii) In the case where $\theta = \frac{1}{3}\pi$, show that the exact length of the perimeter of triangle ABC is $(24 + 8\sqrt{3})$ cm.





The diagram shows a metal plate OABC, consisting of a right-angled triangle OAB and a sector OBC of a circle with centre O. Angle AOB = 0.6 radians, OA = 6 cm and OA is perpendicular to OC.

(i) Show that the length of OB is 7.270 cm, correct to 3 decimal places.

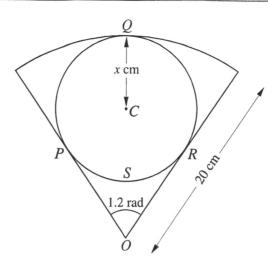
[1]

(ii) Find the perimeter of the metal plate.

[3]

(iii) Find the area of the metal plate.

11



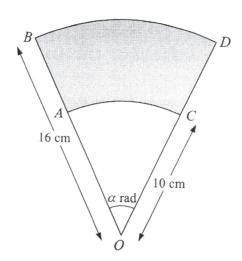


N-12-12-01)

The diagram shows a sector of a circle with centre O and radius 20 cm. A circle with centre C and radius x cm lies within the sector and touches it at P, Q and R. Angle POR = 1.2 radians.

(i) Show that x = 7.218, correct to 3 decimal places.

- [4]
- (ii) Find the total area of the three parts of the sector lying outside the circle with centre C. [2]
- (iii) Find the perimeter of the region OPSR bounded by the arc PSR and the lines OP and OR. [4]





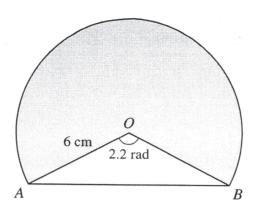
In the diagram, OAB and OCD are radii of a circle, centre O and radius 16 cm. Angle $AOC = \alpha$ radians. AC and BD are arcs of circles, centre O and radii 10 cm and 16 cm respectively.

(i) In the case where $\alpha = 0.8$, find the area of the shaded region.

[2]

(ii) Find the value of α for which the perimeter of the shaded region is 28.9 cm.

N-5 [3]

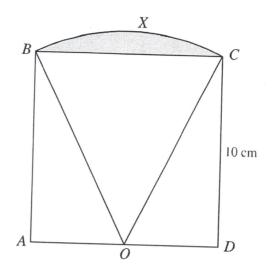




The diagram shows part of a circle with centre O and radius 6 cm. The chord AB is such that angle AOB = 2.2 radians. Calculate

(i) the perimeter of the shaded region,

- [3]
- (ii) the ratio of the area of the shaded region to the area of the triangle AOB, giving your answer in the form k:1.





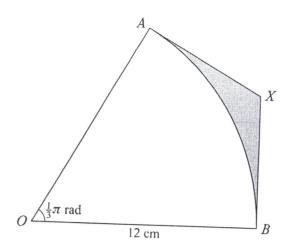
[2]

The diagram shows a square ABCD of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O. 7.13-12.4

- (i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places.
- (ii) Find the perimeter of the shaded region. [3]
- (iii) Find the area of the shaded region. [2]

CIRCULAR MEASURE-AREA OF SECTOR

15



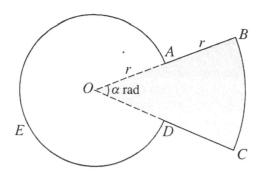


In the diagram, OAB is a sector of a circle with centre O and radius 12 cm. The lines AX and BX are tangents to the circle at A and B respectively. Angle $AOB = \frac{1}{3}\pi$ radians.

(i) Find the exact length of AX, giving your answer in terms of $\sqrt{3}$.

[2]

(ii) Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$.





[3]

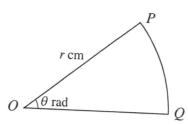
The diagram shows a metal plate made by fixing together two pieces, OABCD (shaded) and OAED (unshaded). The piece OABCD is a minor sector of a circle with centre O and radius 2R. The piece OAED is a major sector of a circle with centre O and radius R. Angle AOD is α radians. Simplifying your answers where possible, find, in terms of α , π and R, N-18-11-(6)

- (i) the perimeter of the metal plate,
- (ii) the area of the metal plate. [3]

It is now given that the shaded and unshaded pieces are equal in area.

(iii) Find α in terms of π .

17





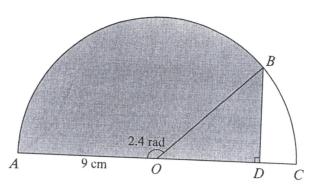
A piece of wire of length 50 cm is bent to form the perimeter of a sector POQ of a circle. The radius of the circle is r cm and the angle POQ is θ radians (see diagram). $\sim 9 - 12 - (7)$

(i) Express θ in terms of r and show that the area, $A \text{ cm}^2$, of the sector is given by

$$A = 25r - r^2. ag{4}$$

(ii) Given that r can vary, find the stationary value of A and determine its nature.

[4]





In the diagram, ABC is a semicircle, centre O and radius 9 cm. The line BD is perpendicular to the diameter AC and angle AOB = 2.4 radians.

J-5

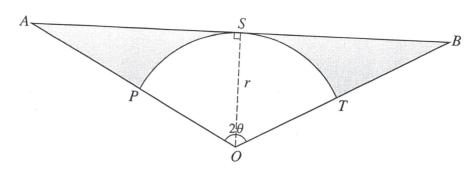
(i) Show that BD = 6.08 cm, correct to 3 significant figures.

[2]

(ii) Find the perimeter of the shaded region.

[3]

(iii) Find the area of the shaded region.



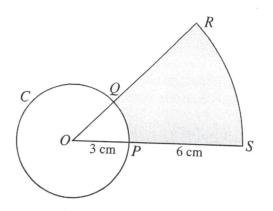
In the diagram, OAB is an isosceles triangle with OA = OB and angle $AOB = 2\theta$ radians. Arc PSThas centre O and radius r, and the line ASB is a tangent to the arc PST at S.

(i) Find the total area of the shaded regions in terms of r and θ .

[4]

(ii) In the case where $\theta = \frac{1}{3}\pi$ and r = 6, find the total perimeter of the shaded regions, leaving your J-11-11-9







The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O. Given that PS = 6 cm and that the area of the shaded region is equal to the area of circle C, $\sqrt{-13-13-2}$

(i) show that angle $POQ = \frac{1}{4}\pi$ radians,

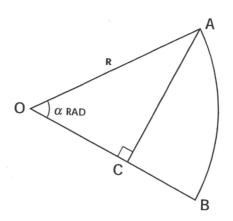
[3]

(11) find the perimeter of the shaded region.

[2]

2.1

\$-15-13-CH





In the diagram, OAB is a sector of a circle with centre O and radius r. The point C on OB is such that angle ACO is a right angle. Angle AOB is α radians and is such that AC divides the sector into two regions of equal area.

(i) Show that $\sin \alpha \cos \alpha = \frac{1}{2}\alpha$.

[4]

It is given that the solution of the equation in part (i) is $\alpha = 0.9477$, correct to 4 decimal places.

(ii) Find the ratio

perimeter of region OAC: perimeter of region ACB,

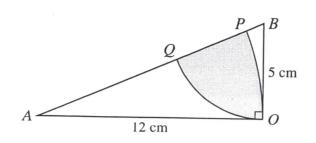
giving your answer in the form \mathbf{k} : 1, where \mathbf{k} is given correct to 1 decimal place.

[5]

(iii) Find angle AOB in degrees.

[1]

22





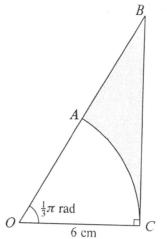
The diagram shows a triangle AOB in which OA is 12 cm, OB is 5 cm and angle AOB is a right angle. Point P lies on AB and OP is an arc of a circle with centre A. Point Q lies on AB and OQ is an arc of a circle with centre B. N-14-12-14

(i) Show that angle BAO is 0.3948 radians, correct to 4 decimal places.

[1]

(ii) Calculate the area of the shaded region.

[5]

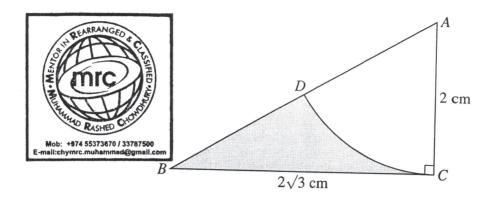




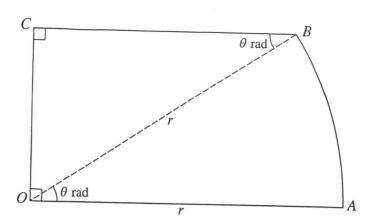
In the diagram, AC is an arc of a circle, centre O and radius 6 cm. The line BC is perpendicular to OC and OAB is a straight line. Angle $AOC = \frac{1}{3}\pi$ radians. Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$.

2-4-2

24



In the diagram, D lies on the side AB of triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC. Find the area of the shaded region BDC, giving your answer in terms of π and $\sqrt{3}$. N-12-13-4[4]





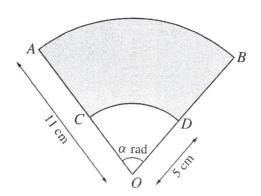
The diagram represents a metal plate OABC, consisting of a sector OAB of a circle with centre O and radius r, together with a triangle OCB which is right-angled at C. Angle $AOB = \theta$ radians and OC is perpendicular to OA.

(i) Find an expression in terms of r and θ for the perimeter of the plate.

[3]

(ii) For the case where r = 10 and $\theta = \frac{1}{5}\pi$, find the area of the plate.



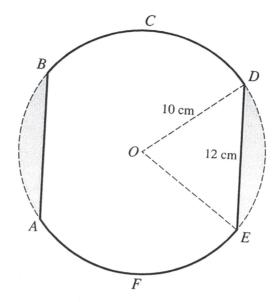


The diagram shows sector OAB with centre O and radius 11 cm. Angle AOB = α radians. Points C and D lie on OA and OB respectively. Arc CD has centre O and radius 5 cm.

- (i) The area of the shaded region ABDC is equal to k times the area of the unshaded region OCD.[3]
- (ii) The perimeter of the shaded region ABDC is equal to twice the perimeter of the unshaded region OCD. Find the exact value of α .

 (ii) The perimeter of the shaded region ABDC is equal to twice the perimeter of the unshaded region α .





The diagram shows a metal plate ABCDEF which has been made by removing the two shaded regions from a circle of radius 10 cm and centre O. The parallel edges AB and ED are both of length 12 cm.

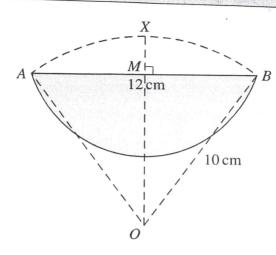
(i) Show that angle *DOE* is 1.287 radians, correct to 4 significant figures.

[2]

(ii) Find the perimeter of the metal plate.

[3]

(iii) Find the area of the metal plate. [3]



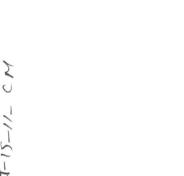


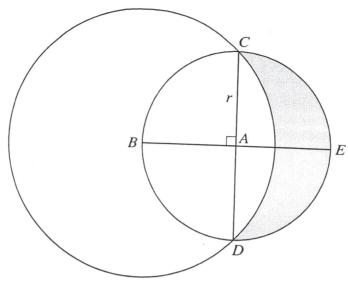
In the diagram, OAXB is a sector of a circle with centre O and radius $10 \, \mathrm{cm}$. The length of the chord AB is $12 \, \mathrm{cm}$. The line OX passes through M, the mid-point of AB, and OX is perpendicular to AB. The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA.

i) Show that angle AXB is 2.498 radians, correct to 3 decimal places.	[3]
	••••••••••
	•••••••••••••••••••••••••••••••••••••••
	•••••
Find the perimeter of the shaded region.	[3]
	[5]
	••••••
	••••••
	•••••••

Find the area of the shaded region.	[3]
	•••••
	••••••••
	Find the perimeter of the shaded region. Find the area of the shaded region.

29







The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(i) Show that the radius of the larger circle is $r\sqrt{2}$.

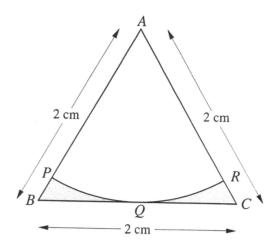
[1]

(ii) Find the area of the shaded region in terms of r.

[6]

30

9-12-11

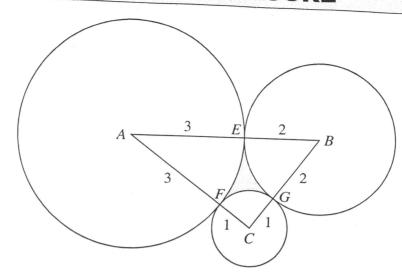




In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q. An arc of a circle with centre A touches BC at Q, and meets AB at P and AC at R. Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. J-(2-11-3)[5]

3 1

2-16-18 CM



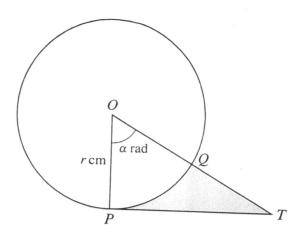
The diagram shows triangle ABC where AB = 5 cm, AC = 4 cm and BC = 3 cm. Three circles with centres at A, B and C have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points E, F and G, lying on AB, AC and BC respectively. Find the area of the shaded region EFG.

[7]



3 2

\$-11-12-CM





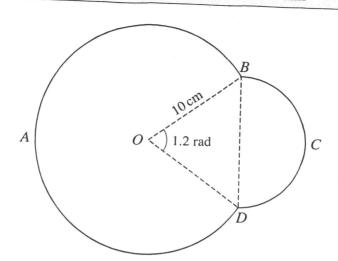
The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α .

- [3]
- (ii) In the case where $\alpha = \frac{1}{3}\pi$ and r = 10, find the area of the shaded region correct to 2 significant figures.

33

7-16-12 CM





The diagram shows a metal plate ABCD made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius $10 \, \mathrm{cm}$. Angle BOD is $1.2 \, \mathrm{radians}$.

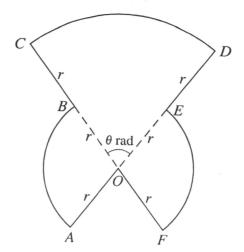
- (i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places.
- [2]

(ii) Find the perimeter of the metal plate.

[3]

(iii) Find the area of the metal plate.

34





71513

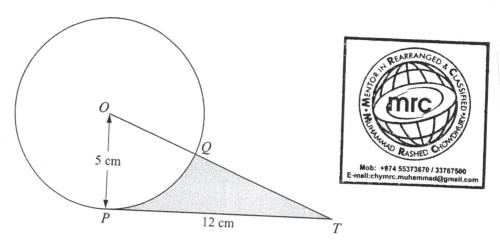
The diagram shows a metal plate OABCDEF consisting of 3 sectors, each with centre O. The radius of sector COD is 2r and angle COD is θ radians. The radius of each of the sectors BOA and FOE is r, and AOED and CBOF are straight lines.

(i) Show that the area of the metal plate is $r^2(\pi + \theta)$.

[3]

(ii) Show that the perimeter of the metal plate is independent of θ .

[4]



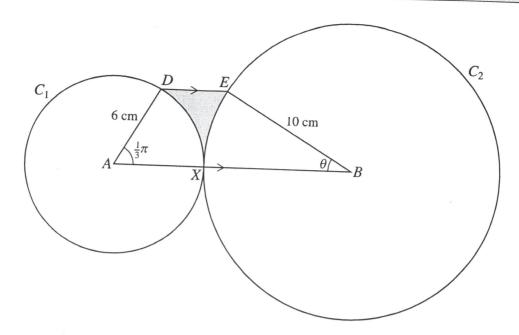
The diagram shows a circle with centre O and radius 5 cm. The point P lies on the circle, PT is a tangent to the circle and PT = 12 cm. The line OT cuts the circle at the point Q.

(i) Find the perimeter of the shaded region.

7-8-(51)

[4]

(ii) Find the area of the shaded region.



The diagram shows a circle C_1 touching a circle C_2 at a point X. Circle C_1 has centre A and radius 6 cm, and circle C_2 has centre B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB. Angle $DAX = \frac{1}{3}\pi$ radians and angle $EBX = \theta$ radians. N - 1/2 - (6/3)

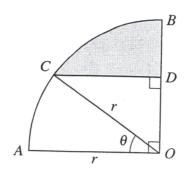
(i) By considering the perpendicular distances of D and E from AB, show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$.

(ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]



37

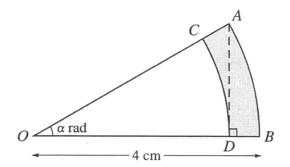




In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to AO and angle $AOC = \theta$ radians.

- (i) Express the perimeter of the shaded region in terms of r, θ and π .
- (ii) For the case where r = 5 cm and $\theta = 0.6$, find the area of the shaded region. [3]

[4]

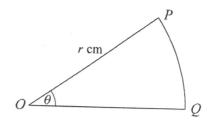




In the diagram, AB is an arc of a circle with centre O and radius 4 cm. Angle AOB is α radians. The point D on OB is such that AD is perpendicular to OB. The arc DC, with centre O, meets OA at C. N-14-11-36

- (i) Find an expression in terms of α for the perimeter of the shaded region ABDC. [4]
- (ii) For the case where $\alpha = \frac{1}{6}\pi$, find the area of the shaded region *ABDC*, giving your answer in the form $k\pi$, where k is a constant to be determined.

39



The diagram shows the sector OPQ of a circle with centre O and radius r cm. The angle POQ is θ radians and the perimeter of the sector is 20 cm.

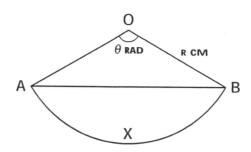
- (i) Show that $\theta = \frac{20}{r} 2$.
- (ii) Hence express the area of the sector in terms of r.
- (iii) In the case where r = 8, find the length of the chord PQ.



- [2]
- [2]
- [3]

40

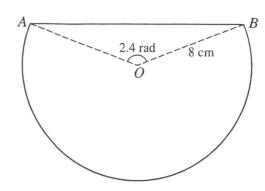




The diagram shows a sector of a circle with radius r cm and centre O. The chord AB divides the sector into a triangle AOB and a segment AXB. Angle AOB is θ radians.

- (1) In the case where the areas of the triangle AOB and the segment AXB are equal, find the value of the constant \mathbf{p} for which $\theta = \mathbf{p} \sin \theta$.
- (II) In the case where r = 8 and $\theta = 2.4$, find the perimeter of the segment AXB. [3]

4 1



The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm. The line AB is a chord of the circle and angle AOB = 2.4 radians. Find

(i) the length of AB,

J-12-12-(G) [2]

(ii) the perimeter of the plate,

[3]

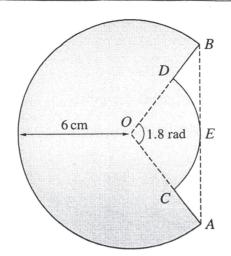
[3]

(iii) the area of the plate.



42

J-16-13-CM

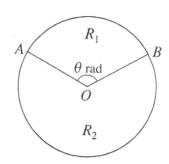




The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O. Angle COD = 1.8 radians.

- (i) Show that the radius of the arc *CED* is 3.73 cm, correct to 3 significant figures. [2]
- (ii) Find the area of the shaded region. [4]

43





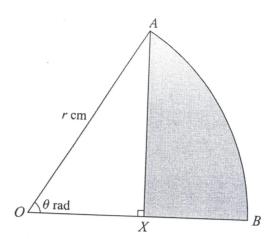
The diagram shows a circle with centre O. The circle is divided into two regions, R_1 and R_2 , by the radii OA and OB, where angle $AOB = \theta$ radians. The perimeter of the region R_1 is equal to the length of the major arc AB.

(i) Show that $\theta = \pi - 1$.

[3]

(ii) Given that the area of region R_1 is $30 \, \mathrm{cm}^2$, find the area of region R_2 , correct to 3 significant figures.

44





In the diagram, AB is an arc of a circle, centre O and radius rcm, and angle $AOB = \theta$ radians. The point X lies on OB and AX is perpendicular to OB.

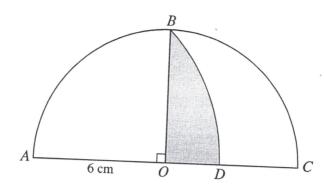
N-7-7

(i) Show that the area, $A \text{ cm}^2$, of the shaded region AXB is given by

$$A = \frac{1}{2}r^2(\theta - \sin\theta\cos\theta).$$
 [3]

(ii) In the case where r = 12 and $\theta = \frac{1}{6}\pi$, find the perimeter of the shaded region AXB, leaving your answer in terms of $\sqrt{3}$ and π .

45





The diagram shows a semicircle ABC with centre O and radius $6\,\mathrm{cm}$. The point B is such that angle BOA is 90° and BD is an arc of a circle with centre A. Find

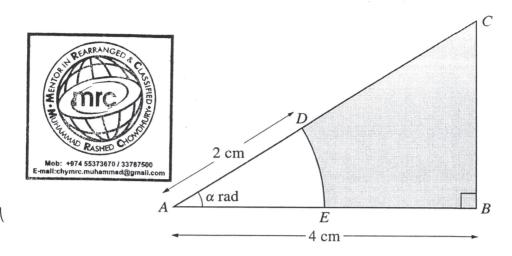
(i) the length of the arc BD,

[4]

(ii) the area of the shaded region.

[3]

46



DY

The diagram shows triangle ABC in which AB is perpendicular to BC. The length of AB is 4 cm and angle CAB is α radians. The arc DE with centre A and radius 2 cm meets AC at D and AB at E. Find, in terms of α , $\mathcal{I}-\mathcal{I}(A-B)$

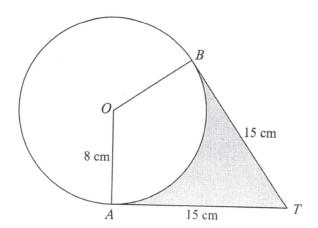
(i) the area of the shaded region,

[3]

(ii) the perimeter of the shaded region.

[3]

47





The diagram shows a circle with centre O and radius 8 cm. Points A and B lie on the circle. The tangents at A and B meet at the point T, and AT = BT = 15 cm.

(i) Show that angle AOB is 2.16 radians, correct to 3 significant figures.

[3]

(ii) Find the perimeter of the shaded region.

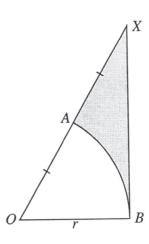
[2]

(iii) Find the area of the shaded region.

[3]

48





In the diagram, AB is an arc of a circle with centre O and radius r. The line XB is a tangent to the circle at B and A is the mid-point of OX. $\sqrt{-12-13}$

(i) Show that angle $AOB = \frac{1}{3}\pi$ radians.

[2]

Express each of the following in terms of r, π and $\sqrt{3}$:

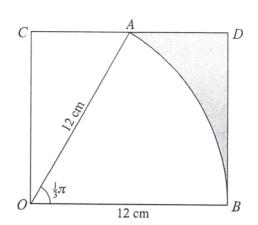
(ii) the perimeter of the shaded region,

[3]

(iii) the area of the shaded region.

[2]

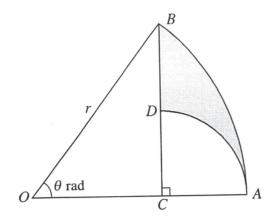
49





In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle OCDB. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b.

5 0





The diagram shows a sector OAB of a circle with centre O and radius r. Angle AOB is θ radians. The point C on OA is such that BC is perpendicular to OA. The point D is on BC and the circular arc AD has centre C.

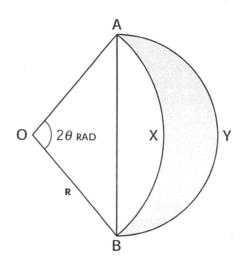
(i) Find AC in terms of r and θ .

[1]

(ii) Find the perimeter of the shaded region ABD when $\theta = \frac{1}{3}\pi$ and r = 4, giving your answer as an exact value. [6]

5 1







In the diagram, AYB is a semicircle with AB as diameter and OAXB is a sector of a circle with centre O and radius r. Angle AOB = 2θ radians. Find an expression, in terms of r and θ , for the area of the shaded region.

52

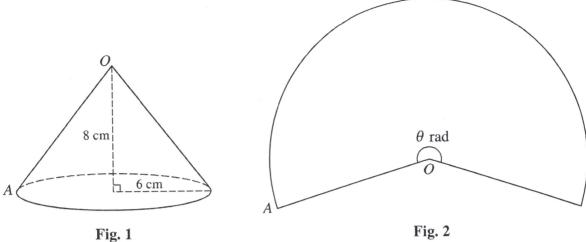


Fig. 1 shows a hollow cone with no base, made of paper. The radius of the cone is 6 cm and the height is 8 cm. The paper is cut from A to O and opened out to form the sector shown in Fig. 2. The circular bottom edge of the cone in Fig. 1 becomes the arc of the sector in Fig. 2. The angle of the sector is N-13-12-2 θ radians. Calculate

(i) the value of θ ,

[4]

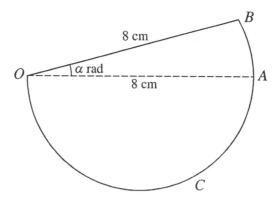
[2]

(ii) the area of paper needed to make the cone.



53





In the diagram, OAB is a sector of a circle with centre O and radius 8 cm. Angle BOA is α radians. OAC is a semicircle with diameter OA. The area of the semicircle OAC is twice the area of the sector OAB. 2-13-11-3

(i) Find α in terms of π .

[3]

(ii) Find the perimeter of the complete figure in terms of π .

[2]